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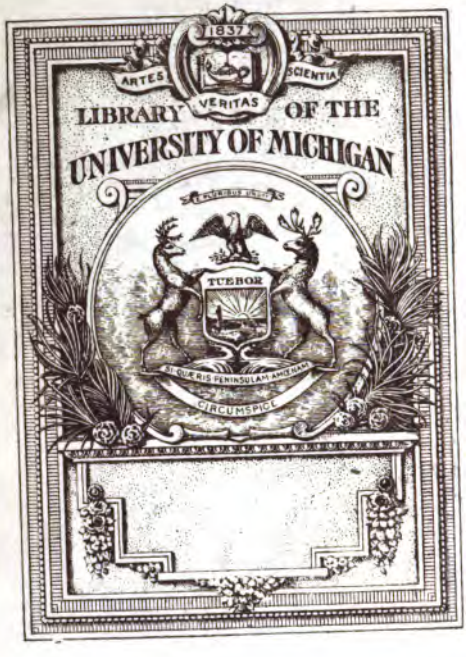

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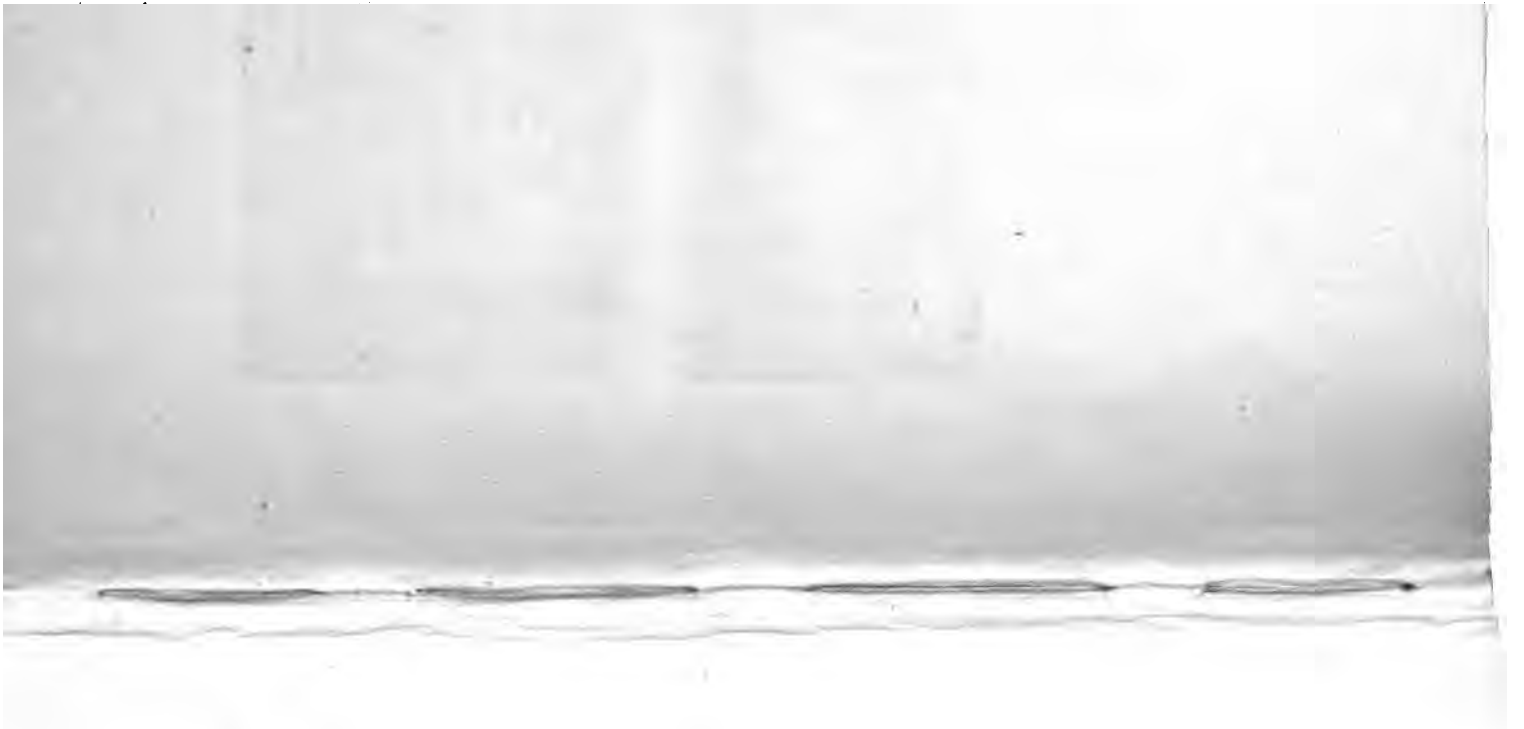

BUILDING
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BUILDING
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H
71
.P5
1755

BUILDING
USE ONL

BUILDING
USE ON

308
P51

Contents

1. Several Essays in Political Arithmetick; by Sir Will. Petty Lond: 1755

2. Annuities on Lives: by Ab: De Moivre — Lond: 1750

3. Valuation of Annuities and Leases for a single Life; by Weyman Lee, Esq. Lond: 1751

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SEVERAL

ESSAYS

IN

Political Arithmetick.

BY

Sir WILLIAM PETTY, Knt.

AND

FELLOW *of the* ROYAL SOCIETY.

The FOURTH EDITION, Corrected.

To which are prefix'd,

MEMOIRS

OF THE

AUTHOR's LIFE.

LONDON:

Printed for D. BROWNE, without *Temple-Bar*; J. SHUCKBURGH, at the *Sun*, and J. WHISTON and B. WHITE, at *Boyle's* Head in *Fleet-Street*.

M.DCC.LV.

THE CONTENTS.

Page

LIFE *of Sir* William Petty —— 1

Eſſay *concerning the Multiplication of Mankind* —— —— 4

Eſſay *on the Growth of the City of* London —— —— —— 9

Obſervations *upon the* Dublin *Bills of Mortality, and the State of that City* —— 35

Farther Obſervations *upon the ſame* —— 51

Two Eſſays *in Political Arithmetick, comparing the Cities of* London *and* Paris 57

Obſervations *on the Cities of* London *and* Rome —— —— 67

Five

Page

Five Eſſays *in Political Arithmetick* —— 71

Political Arithmetick, *concerning the Value of Lands, People, Buildings, Manufactures, Commerce, Fiſhery, Revenues, Taxes, Intereſt, Registries, Banks, Militias, Shiping,* &c. *particularly as the ſame relates to* Great Britain *and its Territories, to* Holland *and* France —— 95

THE

THE

LIFE

OF THE

AUTHOR.

SIR *William Petty*, the ingenious Author of the following judicious Tracts, was Son of *Anthony Petty*, a Clothier at *Rumſey* in *Hampſhire*, and born *May* 26, 1623. He took great Delight, whilſt a Boy, in ſpending his Time among Carpenters, Smiths, and other Artificers, whoſe Trades he ſo well underſtood, that at 12 Years of Age he could work at them. His Education was only at the Grammar School there, but he made ſo great a Progreſs

in his Learning, that when he was 15, he had made himſelf Maſter of *Latin*, *Greek*, and *French*, underſtood *Dialling*, and ſo much of *Geometry* and *Aſtronomy*, as was uſeful to Navigation.

Soon after he went to *Caen* in *Normandy*, with a little Stock of Merchandize, which he there improved: From thence to *Paris*, where he ſtudied *Anatomy*, and came there acquainted with Mr. *Hobbes*, who had a great Affection for him, and aſſiſted him in his Studies. Upon his Return to *England* he had a Place given him in the Royal Navy.

In 1643, when the War grew hot between the King and Parliament, he went into the *Netherlands* and *France*, proſecuting his Studies for 3 Years, and then returned to *Rumſey*, with ſo ſmall a Stock as only about 70 *l.* in Caſh, but with an inexhauſtible Treaſure of uſeful Learning.

In 1647, he obtained a Patent from the Parliament for 17 Years to teach the *Art of double Writing*. He ſided with the Parliament in Opinion, and in 1648 went to *Oxford*, where he became Deputy to Dr. *Tho. Clayton*, Profeſſor of Anatomy, and under him inſtructed young Students in that Science, and in Chemiſtry.

In *March* 1649, he was created Doctor of Phyſick, and ſoon after made Fellow of *Brazen Noſe* College. In *December* 1650, he was one of the Perſons concerned in recovering *Anne Green* to Life, who was hanged at *Oxford* on the 14th for the ſuppoſed Murther of her Baſtard Child. *January* following he was elected Profeſſor of Anatomy in the Room of Dr. *Clayton* who reſigned, and ſoon after was made a Member of the College of Phyſicians, and Muſic-Profeſſor at *Greſham* College.

Being now Maſter of 500*l.* he went to *Ireland*, where he was made Phyſician to the Army by the Parliament, with an Allowance of 20 *s. per Diem*, in which Poſt he continued till *June* 1659, gaining by his Practice 4000 *l. per Annum*, beſide his Salary.

In 1654, perceiving that the Lands which were forfeited in 1641, and given to the Soldiers for ſuppreſſing the Rebellion, then in *Ireland* were very falſly meaſured, and therefore unequally divided by the Unſkillfulneſs of the Surveyors, he entered into a Contract in *December* 1654 to regulate the Admeaſurements, by which he gained 9000*l.* He was then made Clerk of the Council for two Years, with a Salary of 400 *l. per Annum*. By theſe Advantages, and his conſiderable Practice in Phyſick, he had now increaſed his Fortune ſo much as to be worth full 13 Thouſand Pounds; Part of

this Money he improved by purchaſing Soldiers Debentures, and with Part he bought the Earl of *Arundel*'s Houſe in *Lothbury*, *London*. The reſt he kept in Caſh for any improveable Emergency.

January 1658, he was elected a Burgeſs for *Weſtlow* in *Cornwall*, in the Parliament called by *Richard Cromwell*; but that being ſoon diſſolved, he returned to *Ireland*, where he had a great Conteſt with Sir *Jerom Sankey*, who charged him with amaſſing a great Fortune by taking great Bribes, buying Debentures againſt the Statute, by fraudulently procuring ſeveral Lands, having uſed foul Practices in ſetting out the Lands, and other Miſdemeanors; againſt which Charges Mr. *Petty* vindicated himſelf in a ſmall Tract, but it doth not appear that he received any publick Cenſure.

After the Reſtoration he returned to *England*, and was introduced to King *Charles* II. who was ſo highly pleaſed with his Ingenuity, that *April* 11, 1661, he knighted him, and, as it was reported, deſigned to create him Earl of *Kilmore* in *Ireland*. Upon the Eſtabliſhment of the Royal Society, he was one of the firſt Members, and afterwards one of its Council.

In 1663, Sir *William Petty* was greatly applauded in *Ireland* for the Succeſs of his new Invention of a double-bottomed Ship; ſhe ſailed

ſailed from *Dublin* in *July* 1663, and turned into the narrow Harbour of *Holyhead*, amongſt the Rocks and Ships, with ſuch Dexterity, that many experienced Seamen did confeſs they had never ſeen the like. It appeared very much to excell all other Forms of Ships in Sailing, in Carriage, in Security, and many other Benefits. Sir *William* gave a Model of this Ship, made with his own Hand, to the Royal Society, and it is now in their Muſæum; and, as Biſhop *Sprat* obſerves, this Invention will undoubtedly produce great Effects, if ever it ſhall be brought to Perfection, upon the publick Stock of the Nation.

He died at his Houſe in *Picadilly* the 16th of *December* 1687, of a Gangrene in the Foot, occaſioned by the Gout, in his 63d Year, and was buried at his native Town *Rumſey*. By his Wife *Elizabeth*, Daughter of Sir *Hardres Waller*, Knight, and Relict of Sir *Maurice Fenton*, he had Iſſue *Charles*, *Henry*, and one Daughter *Anne*. *Charles* the eldeſt Son was ſoon after his Father's Death created Baron of *Shelburn* in *Ireland*.

By his laſt Will it appears that he eſtimated his real Eſtate at 6500 *l. per Annum*, his perſonal Eſtate about 45000 *l.* and the demonſtrable Improvements of his *Iriſh* Eſtates at 4000 *l. per Annum*; In all he may be reckoned to have left behind him to the Amount of 15000 *l. per*

Annum. A prodigious Fortune to raiſe from ſo ſmall a Beginning.

He wrote ſeveral ſmall Treatiſes, chiefly mathematical, ſome of which are inſerted in the *Philoſophical Tranſactions.* A Catalogue of them is annexed to his Life in the *General Dictionary, Vol.* 8. *Article* PETTY. But the moſt eſteemed, and indeed moſt uſeful, are thoſe which are here collected, and have already gone through three Editions; but being become very ſcarce, and much ſought after, it was thought that a new Edition would be acceptable to the Publick, which is printed from the moſt correct and compleat Editions of his ſeveral Tracts.

AN

ESSAY

Concerning

The MULTIPLICATION of Mankind:

Together with another

ESSAY

IN

POLITICAL ARITHMETICK,

Concerning the Growth of the

CITY OF LONDON:

WITH THE

Meaſures, Periods, Cauſes, and Conſequences thereof, 1682.

The Second Edition, reviſed and enlarged.

By Sir WILLIAM PETTY, *Fellow of the* Royal Society.

THE STATIONER TO THE READER.

THE enſuing Eſſay concerning the Growth of the City of *London*, was intituled [*Another Eſſay*] intimating that ſome other Eſſay had preceded it, which was not to be found. I having been much importuned for that precedent Eſſay, have found that the ſame was about the Growth, Encreaſe, and Multiplication of Mankind, which Subject ſhould in Order of Nature precede that of the Growth of the City of *London*, but am not able to procure the Eſſay itſelf, only I have obtained from a Gentleman, who ſometimes correſponded with Sir *William Petty*, an Extract of a Letter from Sir *William* to him, which I verily believe containeth the Scope thereof; wherefore, I muſt deſire the Reader to be content therewith, till more can be had.

The

The Extract of a Letter concerning the Scope of an Eſſay intended to precede another Eſſay concerning the Growth of the City of London, *&c. An Eſſay in Political Arithmetick, concerning the Value and Encreaſe of People and Colonies.*

THE Scope of this Eſſay, is concerning People and Colonies, and to make way for another Eſſay concerning the Growth of the City of *London*. I deſire in this firſt Eſſay to give the World ſome Light, concerning the Numbers of People in *England*, with *Wales*, and in *Ireland*; as alſo, of the Number of Houſes, and Families, wherein they live, and of Acres they occupy.

2. How many live upon their Lands, how many upon their Perſonal Eſtates, and Commerce, and how many upon Art and Labour; how many upon Alms, how many upon Offices and Publick Employments, and how many as Cheats and Thieves; how many are Impotents, Children, and decrepit old Men.

3. How many upon the Poll-Taxes in *England*, do pay extraordinary Rates, and how many at the Level.

4. How many Men and Women are prolifick, and how many of each are married or unmarried.

5. What the Value of People are in *England*, and what in *Ireland*, at a Medium, both as Members

Members of the Church or Commonwealth, or as Slaves and Servants to one another; with a Method how to eſtimate the ſame, in any other Country or Colony.

6. How to compute the Value of Land in Colonies, in Compariſon to *England* and *Ireland*.

7. How 10000 People in a Colony may be, and planted to the beſt Advantage.

8. A Conjecture in what Number of Years *England* and *Ireland* may be fully peopled, as alſo all *America*, and laſtly the whole habitable Earth.

9. What Spot of the Earth's Globe were fitteſt for a general and univerſal Emporium, whereby all the People thereof may beſt enjoy one anothers Labours and Commodities.

10. Whether the ſpeedy Peopling of the Earth would make

Firſt, For the Good of Mankind.

Secondly, To fulfil the revealed Will of God.

Thirdly, To what Prince or State the ſame would be moſt advantageous.

11. An Exhortation to all thinking Men to ſalve the Scriptures, and other good Hiſtories, concerning the Number of People in all Ages of the World, in the great Cities thereof, and elſewhere.

12. An Appendix concerning the different Number of Sea-fiſh and Wild-fowl, at the End of every thouſand Years, ſince *Noah*'s Flood.

13. An Hypotheſis of the Uſe of thoſe Spaces (of about 8000 Miles through) within the Globe

Globe of our Earth, ſuppoſing a Shell of 150 Miles thick.

14. What may be the Meaning of glorified Bodies, in caſe the Place of the Bleſſed ſhall be without the Convex of the Orb of the fixed Stars, if that the whole Syſtem of the World was made for the Uſe of our Earth's-men.

The principal Points of this Diſcourſe.

1. THAT *London* doubles in Forty Years, and all *England* in Three Hundred and ſixty Years.

2. That there be, *Anno* 1682. about ſix Hundred and ſeventy Thouſand Souls in *London*, and about ſeven Millions four hundred Thouſand in all *England* and *Wales*, and about twenty-eight Millions of Acres of profitable Land.

3. That the Periods of doubling the People, are found to be in all Degrees, from between Ten, to Twelve hundred Years.

4. That the Growth of *London* muſt ſtop of itſelf, before the Year 1800.

5. A Table helping to underſtand the Scriptures, concerning the Number of People mentioned in them.

6. That the World will be fully peopled within the next two Thouſand Years.

7. Twelve Ways whereby to try any Propoſal, pretended for the Publick Good.

8. How

8. How the City of *London* may be made (morally ſpeaking) invincible.

9. An Help to Uniformity in Religion.

10. That it is poſſible to encreaſe Mankind by Generation four Times more than at preſent.

11. The Plagues of *London* are the chief Impediment and Objection, againſt the Growth of the City.

12. That an exact Account of the People is neceſſary in this Matter.

Of the Growth of the City of London: *And of the Meaſures, Periods, Cauſes, and Conſequences thereof.*

BY the City of *London*, we mean the Houſes within the Walls of the old City, with the Liberties thereof, *Weſtminſter*, the Borough of *Southwark*, and ſo much of the built Ground in *Middleſex* and *Surrey*, whoſe Houſes are contiguous unto, or within Call of thoſe afore-mentioned. Or elſe we mean the Houſes which ſtand upon the Ninety ſeven Pariſhes within the Walls of *London*; upon the Sixteen Pariſhes next, without them; the Six Pariſhes of *Weſtminſter*, and the Fourteen out-Pariſhes in *Middleſex* and *Surrey*, contiguous to the former; all which One hundred and thirty three Pariſhes are comprehended within the Weekly Bills of Mortality.

The Growth of this City is meaſured; 1. By the Quantity of Ground, or Number of Acres upon which it ſtands. 2. By the Number of Houſes, as the ſame appears by the Hearth-books and late Maps. 3. By the Cubical Content of the ſaid Houſes. 4. By the Flooring of the ſame. 5. By the Number of Days-work, or Charge of building the ſaid Houſes. 6. By the Value of the ſaid Houſes, according to their yearly Rent, and Number of Years Purchaſe. 7. By the Number of Inhabitants; according to which latter

latter Senſe only, we make our Computations in this Eſſay.

Till a better Rule can be obtained, we conceive that the Proportion of the People may be ſufficiently meaſured by the Proportion of the Burials in ſuch Years as were neither remarkable for extraordinary Healthfulneſs or Sicklineſs.

That the City hath increaſed in this latter Senſe, appears from the Bills of Mortality, repreſented in the two following Tables, *viz.* One whereof is a Continuation for Eighteen Years, ending in 1682. of that Table which was publiſhed in the 117th Page of the Book of the *Obſervations upon the* London *Bills of Mortality*, printed in the Year 1676. The other ſheweth what Number of People dyed at a Medium of two Years, indifferently taken, at about Twenty Years Diſtance from each other.

The

The first of the said two Tables.

An. Dom.	97 Parishes.	16 Parishes.	Out-Parishes.	Buried in all.	Besides of the Plague.	Christen'd.
1665	5320	12463	10925	28708	68596	9967
1666	1689	3969	5082	10740	1998	8997
1667	761	6405	8641	15807	35	10938
1668	796	6865	9603	17267	14	11633
1669	1323	7500	10440	19263	3	12335
1670	1890	7808	10500	20198		11997
1671	1723	5938	8063	15724	5	12510
1672	2237	6788	9200	18225	5	12593
1673	2307	6302	8890	17499	5	11895
1674	2801	7522	10875	21198	3	11851
1675	2555	5986	8702	17243	1	11775
1676	2756	6508	9466	18730	2	12399
1677	2817	6632	9616	19065	2	12626
1678	3060	6705	10908	20673	5	12601
1679	3074	7481	11173	21728	2	12288
1680	3076	7066	10911	21053		12747
1681	3669	8136	12166	23971		13355
1682	2975	7009	10707	20691		12653

According to which latter Table, there died as followeth.

The latter of the said two Tables.

There died in *London*, at a Medium between the Years.

1604 and 1605 ——— 5135. A.
1621 and 1622 ——— 8527. B.
1641 and 1642 ——— 11883. C.
1661 and 1662 ——— 15148. D.
1681 and 1682 ——— 22331. E.

Wherein obſerve, That the Number C. is double to A. and 806 over. That D. is double to B. within 1906. That C. and D. is double to A. B. within 293. That E. is double to C. within 1435. That D. and E. is double to B. and C. within 3341. And that C. and D. and E. are double to A. and B. and C. within 1736. And that E. is above Quadruple to A. All which Differences (every way conſidered) do allow the doubling of the People of *London* in forty Years, to be a ſufficient Eſtimate thereof in round Numbers, and without the Trouble of Fractions. We alſo ſay, That 669930 is near the Number of People now in *London*, becauſe the Burials are 22331. which multiplied by 30, (one dying yearly out of 30, as appears in the 94th Page of the aforementioned Obſervations) maketh the ſaid Number; and becauſe there are 84 Thouſand tenanted Houſes (as we are credibly informed) which at 8 in each, makes 672 Thouſand Souls; the ſaid two Accounts differing inconſiderably from each other.

We have thus pretty well found out in what Number of Years (*viz.* in about 40.) that the City of *London* hath doubled, and the preſent Number of Inhabitants to be about 670 Thouſand. We muſt now alſo endeavour the ſame for the whole Territory of *England* and *Wales*. In Order whereunto, we

Firſt ſay, That the Aſſeſſment of *London* is about an eleventh Part of the whole Territory, and therefore that the People of the whole may

may well be eleven Times that of *London*, *viz.* about 7 Millions, 369 Thouſand Souls; with which Account that of the Poll-Money, Hearth-Money, and the Biſhops late Numbering of the Communicants, do pretty well agree; wherefore although the ſaid Number of 7 Millions, 369 Thouſand, be not (as it cannot be) a demonſtrated Truth, yet it will ſerve for a good Suppoſition, which is as much as we want at preſent.

As for the Time in which the People double, it is yet more hard to be found: For we have good Experience (in the ſaid 94th Page of the afore-mentioned Obſervations) That in the Country, but one of Fifty die *per Annum*; and by other late Accounts, that there have been ſometimes but 24 Births for 23 Burials: The which two Points, if they were univerſally, and conſtantly true, there would be Colour enough to ſay, that the People doubled but in about 1200 Years. As for Example: Suppoſe there be 600 People, of which let a fiftieth Part die *per Annum*, then there ſhall die 12 *per Annum*; and if the Births be as 24 to 23, then the Increaſe of the People ſhall be ſomewhat above half a Man *per Annum*, and conſequently the ſuppoſed Number of 600, cannot be doubled but in 1126 Years, which to reckon in round Numbers, and for that the aforementioned Fractions were not exact, we had rather call 1200.

There are alſo other good Obſervations, That even in the Country, one in about 30, or 32 *per Annum*, hath died, and that there have been five Births for four Burials. Now, according to this Doctrine, 20 will die *per Annum* out of the above 600, and 25 will be born, ſo as the Increaſe will be 5, which is a hundred and twentieth Part of the ſaid 600. So as we have two fair Computations, differing from each other as one to ten; and there are alſo ſeveral other good Obſervations for other Meaſures.

I might here inſert, That although the Births in this laſt Computation be 25 of 600, or a Twenty-fourth Part of the People; yet that in natural Poſſibility, they may be near thrice as many, and near 75. For that by ſome late Obſervations, the teeming Females between 15 and 44, are about 180 of the ſaid 600, and the Males of between 18 and 59, are about 180 alſo, and that every teeming Woman can bear a Child once in two Years; from all which it is plain, that the Births may be 90, (and abating 15 for Sickneſs, young Abortions, and natural Barreneſs) there may remain 75 Births, which is an Eighth of the People; which by ſome Obſervations we have found to be but a Two and thirtieth Part, or but a Quarter of what is thus ſhewn to be naturally poſſible. Now, according to this Reckoning, if the Births may be 75 of 600, and the Burials but 15, then the annual Increaſe of the People will be 60; and ſo the ſaid 600 People may double in 10 Years,

Years, which differs yet more from 1200 above-mentioned. Now to get out of this Difficulty, and to temper thoſe vaſt Diſagreements, I took the Medium of 50 and 30 dying *per Annum*, and pitch'd upon 40; and I alſo took the Medium between 24 Births and 23 Burials, and 5 Births for 4 Burials, *viz*, allowing about 10 Births for 9 Burials; upon which Suppoſition there muſt die 15 *per Annum*, out of the above-mentioned 600, and the Births muſt be 16 and two Thirds, and the Increaſe 1, and two Thirds, or five Thirds of a Man; which Number compared with 1800 Thirds, or 600 Men, gives 360 Years for the Time of doubling (including ſome Allowance for Wars, Plagues, and Famine, the Effects whereof, though they be terrible at the Times and Places where they happen, yet in a Period of 360 Years, is no great Matter in the whole Nation.) For the Plagues of *England* in 20 Years hath carried away ſcarce an Eightieth Part of the whole Nation; and the late 10 Years Civil Wars, (the like whereof hath not been in ſeveral Ages before) did not take away above a fortieth Part of the whole People.

According to which Account or Meaſure of doubling, if there be now in *England* and *Wales*, 7 Millions 400 Thouſand People, there were about 5 Millions 526 Thouſand in the Beginning of Queen *Elizabeth*'s Reign, *Anno* 1560, and about two Millions at the *Norman* Conqueſt, of which conſult the *Doomſday-Book*,

and my Lord *Hale*'s *Origination of Mankind.*

Memorandum, That if the People double in 360 Years, that the preſent 320 Millions computed by ſome learned Men, (from the Meaſures of all the Nations of the World, their Degrees of being peopled, and good Accounts of the People in ſeveral of them) to be now upon the Face of the Earth, will within the next 2000 Years, ſo increaſe, as to give one Head for every two Acres of Land in the habitable Part of the Earth. And then, according to the Prediction of the Scriptures, there muſt be Wars and great Slaughter, *&c.*

Wherefore, as an Expedient againſt the above-mentioned Difference between 10 and 17 hundred years, we do for the preſent, and in this Country admit of 360 years to be the Time wherein the People of *England* do double, according to the preſent Laws and Practice of Marriages.

Now, if the City double its People in 40 years, and the preſent Number be 670 Thouſand, and if the whole Territory be 7 Millions 400 Thouſand, and double in 360 years, as aforeſaid; then by the underwritten Table it appears, that *Anno* 1840, the People of the City will be 10718880, and thoſe of the whole Country but 10917389, which is but inconſiderably more. Wherefore it is certain and neceſſary that the Growth of the City muſt ſtop before the ſaid Year 1840; and will be at its utmoſt

utmoſt Height in the next preceeding Period, *Anno* 1800, when the Number of the City will be Eight times its preſent Number, *viz.* 5 Millions 359 Thouſand. And when (beſides the ſaid Number) there will be 4 Millions 466 Thouſand to perform the Tillage, Paſturage, and other rural Works neceſſary to be done without the ſaid City, as by the following Table, *viz.*

	Annis.	Burials.	People in *London.*	People in *England.*
	1565	2568	77040	5526929.
As in the former Table.	1605	5135		
	1642	11883		
	1682	22331	669930	7369230.
	1722	44662		
	1762	89324		
	1802	178648	5359440	9825650.
	1842	357296	10718889	10917389.

Now when the People of *London* ſhall come to be ſo near the People of all *England*, then it follows, that the Growth of *London* muſt ſtop before the ſaid Year 1842, as aforeſaid, and muſt be at its greateſt Height *Anno* 1800, when it will be eight Times more than now, with above 4 Millions for the Service of the Country and Ports, as aforeſaid.

Of the afore-mentioned vaſt Difference between 10 Years and 1200 Years for doubling the People, we make this Uſe, *viz.* To juſtify the Scriptures and all other good Hiſtories concerning the Number of the People in ancient Time. For ſuppoſing the eight Perſons who

came out of the Ark, increaſed by a progreſſive doubling in every 10 Years, might grow in the firſt 100 Years after the Flood from 8 to 8000, and that in 350 Years after the Flood (when abouts *Noah* died) to one Million, and by this Time 1682, to 320 Millions (which by rational Conjecture, are thought to be now in the World) it will not be hard to compute, how in the intermediate years, the Growths may be made, according to what is ſet down in the following Table, wherein making the doubling to be 10 Years at firſt, and within 1200 Years at laſt, we take a diſcretionary Liberty, but juſtifiable by Obſervations and the Scriptures for the reſt; which Table we leave to be corrected by Hiſtorians, who know the Bigneſs of ancient Cities, Armies, and Colonies in the reſpective Ages of the World, in the mean Time affirming that without ſuch Difference in the Meaſures and Periods for doubling (the Extreams whereof we have demonſtrated to be real and true) it is impoſſible to ſolve what is written in the Holy Scriptures and other authentick Books. For if we pitch upon any one Number throughout for this Purpoſe, 150 Years is the fitteſt of all round Numbers; according to which, there would have been but 512 Souls in the whole World in *Moſes*'s Time (being 800 Years after the Flood) when 603 Thouſand *Iſraelites* of above 20 Years Old (beſides thoſe of other Ages, Tribes, and Nations) were found upon an exact Survey appointed by God,

God, whereas our Table makes 12 Millions. And there would have been but 8000 in *David*'s Time, when were found 1100 Thouſand of above 20 Years old (beſides others, as aforeſaid) in *Iſrael*, upon the Survey inſtigated by Satan, whereas our Table makes 32 Millions. And there would have been but a Quarter of a Million about the Birth of Chriſt, or *Auguſtus* his Time, when *Rome* and the *Roman* Empire were ſo great, whereas our Table makes 100 Millions. Where note, that the *Iſraelites* in about 500 Years between their coming out of *Egypt* to *David's* Reign, increaſed from 603 Thouſand to 1100 Thouſand.

On the other Hand, if we pitch upon a leſs Number, as 100 Years, the World would have been over-peopled 700 Years ſince. Wherefore no one Number will ſolve the Phænomena, and therefore we have ſuppoſed ſeveral in Order to make the following Table, which we again deſire Hiſtorians to correct, according to what they find in Antiquity concerning the Number of the People in each Age and Country of the World.

We did (not long ſince) aſſiſt a worthy Divine, writing againſt ſome Scepticks, who would have baffled our Belief of the Reſurrection, by ſaying, that the whole Globe of the Earth could not furniſh Matter enough for all the Bodies that muſt riſe at the laſt Day, much leſs would the Surface of the Earth furniſh Footing for ſo vaſt a Number; whereas we did

(by

(by the Method aforementioned) aſſert the Number of Men now living, and alſo of thoſe that had died ſince the Beginning of the World, and did withal ſhew, that half the Iſland of *Ireland* would afford them all, not only Footing to ſtand upon, but Graves to lie down in, for that whole Number; and that two Mountains in that Country were as weighty as all the Bodies that had ever been from the Beginning of the World to the Year 1680, when this Diſpute happened. For which Purpoſe, I have digreſſed from my intended Purpoſe, to inſert this Matter, intending to proſecute this Hint further, upon ſome more proper Occaſion.

A Table

A Table ſhewing how the People might have doubled in the ſeveral Ages of the World.

Anno after the Flood.

Periods of doubling,			
	1		8 Perſons.
	10		16
	20		32
	30		64
	40		128
In 10 Years.	50		256
	60		512
	70		1024
	80		2048
	90		4096
	100		8000 and more.
	120 Years after the Flood.		16 Thouſand.
In 20 Years.	140		32
	170		64
30			
	200		128
40	240		256
50	290		512
60	350		1 Million and more.
70	420		2 Millions.
100	520		4 Millions.
190	710		8 Millions.
290	1000		16 In Moſes Time.
400	1400		32 About David's Time.
550	1950		64
750	2700		128 About the Birth of Chriſt.
1000	3700		256
In 300 — 1000	4000		320

It is here to be noted, that in this Table we have aſſigned a different Number of Years for the Time of doubling the People in the ſeveral Ages of the World, and might have done the ſame

ſame for the ſeveral Countries of the World, and therefore the ſaid ſeveral Periods aſſigned to the whole World in the Lump, may well enough conſiſt with the 360 Years eſpecially aſſigned to *England*, between this Day, and the *Norman* Conqueſt; and the ſaid 360 Years may well enough ſerve for a Suppoſition between this Time, and that of the Worlds being fully peopled; nor do we lay any Streſs upon one or the other in this Diſquiſition concerning the Growth of the City of *London*.

We have ſpoken of the Growth of *London*, with the Meaſures and Periods thereof, we come next to the Cauſes and Conſequences of the ſame.

The Cauſes of its Growth from 1642 to 1682, may be ſaid to have been as followeth, *viz.* From 1642 to 1650, that Men came out of the Country to *London*, to ſhelter themſelves from the Outrages of the Civil Wars, during that Time; from 1650 to 1660, the Royal Party came to *London*, for their more private and inexpenſive Living; from 1660 to 1670, the King's Friends and Party came to receive his Favours after his happy Reſtauration; from 1670 to 1680, the Frequency of Plots and Parliaments, might bring extraordinary Numbers to the City; but what Reaſons to aſſign for the like Increaſe from 1604 to 1642, I know not, unleſs I ſhould pick out ſome remarkable Accident happening in each Part of the ſaid Period, and make that to be the Cauſe of this Increaſe

(as

(as vulgar People make the Cauſe of every Man's Sickneſs, to be what he did laſt eat) wherefore, rather than ſo to ſay *quidlibet de quolibet*; I had rather quit even what I have above-ſaid to be the Cauſe of *London*'s Increaſe from 1642 to 1682, and put the whole upon ſome natural and ſpontaneous Benefits and Advantages that Men find by living in great more than in ſmall Societies; and ſhall therefore ſeek for the antecedent Cauſes of this Growth, in the Conſequences of the like, conſidered in greater Characters and Proportions.

Now, whereas in Arithmetick, out of two falſe Poſitions the Truth is extracted, ſo I hope out of two extravagant contrary Suppoſitions, to draw forth ſome ſolid and conſiſtent Concluſion, *viz.*

The firſt of the ſaid two Suppoſitions is, that the City of *London* is ſeven Times bigger than now, and that the Inhabitants of it are four Millions 690 Thouſand People, and that in all the other Cities, Ports, Towns, and Villages, there are but 2 Millions 710 Thouſand Men.

The other Suppoſition is, that the City of *London* is but a ſeventh Part of its preſent Bigneſs, and that the Inhabitants of it are but 96 Thouſand, and that the reſt of the Inhabitants (being 7 Millions 304 Thouſand) do co-habit thus, 104 Thouſand of them in ſmall Cities and Towns, and that the reſt, being 7 Millions 200 Thouſand, do inhabit in Houſes not contiguous to one another, *viz.* In 1200 Thouſand Houſes,

Houſes, having about 24 Acres of Ground belonging to each of them, accounting about 28 Millions of Acres to be in the whole Territory of *England*, *Wales*, and the adjacent Iſlands; which any Man that pleaſes may examine upon a good Map.

Now, the Queſtion is, in which of theſe two imaginary States, would be the moſt convenient, commodious, and comfortable Livings?

But this general Queſtion divides itſelf into the ſeveral Queſtions, relating to the following Particulars, *viz.*

1. For the Defence of the Kingdom againſt foreign Powers.

2. For preventing the inteſtine Commotions of Parties and Factions.

3. For Peace and Uniformity in Religion.

4. For the Adminiſtration of Juſtice.

5. For the proportionably taxing of the People, and eaſy levying the ſame.

6. For Gain by foreign Commerce.

7. For Huſbandry, Manufacture, and for Arts of Delight and Ornament.

8. For leſſening the Fatigue of Carriages and Travelling.

9. For preventing Beggars and Thieves.

10. For the Advancement and Propagation of uſeful Learning.

11. For increaſing the People by Generation.

12. For preventing the Miſchiefs of Plagues and Contagions. And withal, which of the ſaid

ſaid two States is moſt practicable and natural; for in theſe and the like Particulars, do lie the Teſts and Touch-ſtones of all Propoſals, that can be made for the publick Good.

Firſt, as to practicable, we ſay, that although our ſaid extravagant Propoſals are both in Nature poſſible, yet it is not obvious to every Man to conceive, how *London*, now ſeven Times bigger than in the Beginning of Queen *Elizabeth*'s Reign, ſhould be ſeven Times bigger than now it is, and 49 Times bigger than *Anno* 1560. To which I ſay, 1. That the preſent City of *London* ſtands upon leſs than 2500 Acres of Ground, wherefore a City ſeven Times as large may ſtand upon 10500 Acres, which is about equivalent to a Circle of four Miles and a half in Diameter, and leſs than 15 Miles in Circumference. 2. That a Circle of Ground of 35 Miles Semidiameter will bear Corn, Garden-ſtuff, Fruits, Hay, and Timber for the four Millions 690 Thouſand Inhabitants of the ſaid City and Circle, ſo as nothing of that Kind need be brought from above 35 Miles Diſtance from the ſaid City; for the Number of Acres within the ſaid Circle, reckoning two Acres ſufficient to furniſh Bread and Drink-Corn for every Head, and two Acres will furniſh Hay for every neceſſary Horſe; and that the Trees which may grow in the Hedge-rows of the Fields within the ſaid Circle, may furniſh Timber for 600 Thouſand Houſes. 3. That all live Cattle and great Animals can bring them-

themſelves to the ſaid City; and that Fiſh can be brought from the Land's End, and *Berwick*, as eaſily as now. 4. Of Coals there is no Doubt: And for Water, 20 *s. per* Family (or 600 Thouſand Pounds *per Annum* in the whole) will ſerve this City, eſpecially with the Help of the *New-River*. But if by practicable be underſtood, that the preſent State may be ſuddenly changed into either of the two above-mentioned Propoſals, I think it is not practicable. Wherefore the true Queſtion is, unto or towards which of the ſaid two extravagant States it is beſt to bend the preſent State by Degrees, *viz.* Whether it be beſt to leſſen or enlarge the preſent City? In Order whereunto, we enquire (as to the firſt Queſtion) which State is moſt defenſible againſt foreign Powers, ſaying, that if the above-mentioned Houſing, and a Border of Ground, of 3 Quarters of a Mile broad, were encompaſſed with a Wall and Ditch of 20 Miles about (as ſtrong as any in *Europe*, which would coſt but a Million, or about a Penny in the Shilling of the Houſe-Rent for one Year) what foreign Prince could bring an Army from beyond Seas, able to beat, 1. Our Sea-Forces, and next with Horſe harraſſed at Sea, to reſiſt all the freſh Horſe that *England* could make, and then conquer above a Million of Men, well united, diſciplin'd, and guarded within ſuch a Wall, diſtant every-where 3 Quarters of a Mile from the Houſing, to elude the Granadoes and great Shot of the Enemy? 2. As to

in-

inteſtine Parties and Factions, I ſuppoſe that 4 Millions 690 Thouſand People united within this great City, could eaſily govern half the ſaid Number ſcattered without it, and that a few Men in Arms within the ſaid City and Wall, could alſo eaſily govern the reſt unarmed, or armed in ſuch a Manner as the Sovereign ſhall think fit. 3. As to Uniformity in Religion, I conceive, that if St. *Martins* Pariſh may (as it doth) conſiſt of about 40 Thouſand Souls, that this great City alſo may as well be made but as one Pariſh, with 7 Times 130 Chapels, in which might not only be an Uniformity of Common Prayer, but in Preaching alſo; for that a thouſand Copies of one judiciouſly and authentically compoſed Sermon, might be every Week read in each of the ſaid Chapels without any ſubſequent Repetition of the ſame, as in the Caſe of Homilies. Whereas in *England* (wherein are near 10 Thouſand Pariſhes, in each of which upon Sundays, Holy-days, and other extraordinary Occaſions, there ſhould be about 100 Sermons *per Annum*, making about a Million of Sermons *per Annum* in the whole:) It were a Miracle, if a Million of Sermons compoſed by ſo many Men, and of ſo many Minds and Methods, ſhould produce Uniformity upon the diſcompoſed Underſtandings of about 8 Millions of Hearers.

4. As to the Adminiſtration of Juſtice. If in this great City ſhall dwell the Owners of all the Lands, and other valuable Things in *Eng-*

land; if within it ſhall be all the Traders, and all the Courts, Offices, Records, Juries, and Witneſſes; then it follows, that Juſtice may be done with Speed and Eaſe.

5. As to the Equality and eaſy levying of Taxes. It is too certain, that *London* hath at ſome Times paid near half the Exciſe of *England*; and that the People pay thrice as much for the Hearths in *London* as thoſe in the Country, in Proportion to the People of each, and that the Charges of collecting theſe Duties have been about a ſixth Part of the Duty itſelf. Now, in this great City, the Exciſe alone, according to the preſent Laws, would not only be double to the whole Kingdom, but alſo more equal. And the Duty of Hearths of the ſaid City, would exceed the preſent Proceed of the whole Kingdom. And as for the Cuſtoms, we mention them not at preſent.

6. Whether more would be gain'd by foreign Commerce?

The Gain which *England* makes by Lead, Coals, the Freight of Shipping, *&c.* may be the ſame, for aught I ſee, in both Caſes. But the Gain which is made by Manufactures, will be greater, as the Manufacture itſelf is greater and better. For in ſo vaſt a City Manufactures will beget one another, and each Manufacture will be divided into as many Parts as poſſible, whereby the Work of each Artiſan will be ſimple and eaſy: As for Example; in the making of a Watch, if one Man ſhall make the Wheels,

Wheels, another the Spring, another ſhall engrave the Dial-plate, and another ſhall make the Caſes, then the Watch will be better and cheaper, than if the whole Work be put upon any one Man. And we alſo ſee that in Towns, and in the Streets of a great Town, where all the Inhabitants are almoſt of one Trade, the Commodity peculiar to thoſe Places is made better and cheaper than elſewhere. Moreover, when all Sorts of Manufactures are made in one Place, there every Ship that goeth forth, can ſuddenly have its Loading of ſo many ſeveral Particulars and Species, as the Port whereunto ſhe is bound can take off. Again, when the ſeveral Manufactures are made in one Place, and ſhipped off in another, the Carriage, Poſtage, and Travelling-charges, will inhance the Price of ſuch Manufacture, and leſſen the Gain upon foreign Commerce. And laſtly, when the imported Goods are ſpent in the Port itſelf where they are landed, the Carriage of the ſame into other Places, will create no Surcharge upon ſuch Commodity; all which Particulars tend to the greater Gain by foreign Commerce.

7. As for Arts of Delight and Ornament.

They are beſt promoted by the greateſt Number of Emulators. And it is more likely that one ingenious curious Man may rather be found out amongſt 4 Millions than 400 Perſons. But as for Huſbandry, *viz.* Tillage and Paſturage, I ſee no Reaſon, but the ſecond State (when each Family is charged with the Culture of about 24 Acres) will beſt promote the ſame.

8. As for leſſening the Fatigue of Carriage and Travelling.

The Thing ſpeaks itſelf; for if all the Men of Buſineſs, and all Artiſans do live within five Miles of each other; and if thoſe who live without the great City, do ſpend only ſuch Commodities as grow where they live, then the Charge of Carriage and Travelling could be little.

9. As to the preventing of Beggars and Thieves.

I do not find how the Differences of the ſaid two States ſhould make much Difference in this Particular; for Impotents (which are but one in about 600) ought to be maintained by the reſt. 2. Thoſe who are unable to work, through the evil Education of their Parents, ought (for aught I know) to be maintained by their neareſt Kindred, as a juſt Puniſhment upon them. 3. And thoſe who cannot find Work (though able and willing to perform it) by Reaſon of the unequal Application of Hands to Lands, ought to be provided for by the Magiſtrate and Landlord till that can be done; for there need be no Beggars in Countries, where there are many Acres of unimproved improvable Land to every Head, as there are in *England*. As for Thieves, they are for the moſt Part begotten from the ſame Cauſe; for it is againſt Nature, that any Man ſhould venture his Life, Limb, or Liberty, for a wretched Livelihood, whereas moderate Labour will produce a better.

But

But of this ſee Sir *Thomas Moore*, in the firſt Part of his *Utopia*.

10. As to the Propagation and Improvement of uſeful Learning.

The ſame may be ſaid concerning it, as was above-ſaid concerning Manufactures, and the Arts of Delight and Ornaments; for in the great vaſt City, there can be no ſo odd a Conceit or Deſign, whereunto ſome Aſſiſtance may not be found, which in the thin ſcattered Way of Habitation may not be.

11. As for the Increaſe of People by Generation.

I ſee no great Difference from either of the two States, for the ſame may be hindred or promoted in either, from the ſame Cauſes.

12. As to the Plague.

It is to be remembered, that one Time with another, a Plague happeneth in *London* once in 20 Years, or thereabouts; for in the laſt hundred Years, between the Years 1582 and 1682, there have been five great Plagues, *viz.* *Anno* 1592, 1603, 1625, 1636, and 1665. And it is alſo to be remembered, that the Plagues of *London* do commonly kill one fifth Part of the Inhabitants. Now, if the whole People of *England* do double but in 360 Years, then the annual Increaſe of the ſame is but 20000, and in 20 Years 400000. But if in the City of *London* there ſhould be two Millions of People (as there will be about 60 Years hence) then the Plague (killing one Fifth of them, namely,

400000, once in 20 Years) will deſtroy as many in one Year, as the whole Nation can refurniſh in 20: And conſequently the People of the Nation ſhall never increaſe. But if the People of *London* ſhall be above 4 Millions (as in the firſt of our two Extravagant Suppoſitions is premiſed) then the People of the whole Nation ſhall leſſen above 20000 *per Annum*. So as if People be worth 70 *l. per* Head (as hath elſewhere been ſhewn) then the ſaid Greatneſs of the City will be a Damage to itſelf and the whole Nation of 14 hundred Thouſand Pounds *per Annum*, and ſo *pro rata*, for a greater or leſſer Number; wherefore to determine, which of the two States is beſt, (that is to ſay, towards which of the ſaid two States Authority ſhould bend the preſent State) a juſt Balance ought to be made between the Diſadvantages from the Plague, with the Advantages accruing from the other Particulars above-mentioned; unto which Balance a more exact Account of the People, and a better Rule for the Meaſure of its Growth is neceſſary, than what we have here given, or are yet able to lay down.

POST-

POSTSCRIPT.

IT was not very pertinent to a Diſcourſe concerning the City of *London*, to thruſt in Conſiderations of the Time when the whole World will be fully peopled; and how to juſtify the Scriptures concerning the Number of People mentioned in them; and concerning the Number of the Quick and the Dead, that may riſe at the laſt Day, *&c.* Nevertheleſs, ſince ſome Friends liking the ſaid Digreſſions and Impertinencies (perhaps as Sauce to a dry Diſcourſe) have deſired that the ſame might be explained and made out: I therefore ſay as followeth.

1, If the Number of Acres in the habitable Part of the Earth, be under 50 Thouſand Millions; if twenty Thouſand Millions of People, are more than the ſaid Number of Acres will feed; (few or no Countries being ſo fully peopled) and for that in ſix Doublings (which will be in 2000 Years) the preſent 320 Millions will exceed the ſaid 20 Thouſand Millions.

2. That the Number of all thoſe who have died ſince the Flood, is the Sum of all the Products made by multiplying the Number of the doubling Periods mentioned in the firſt Column of the laſt Table, by the Number of People reſpectively affixed to them, in the third Column of the ſame Table; the ſaid Sum being divided by 40 (one dying out of 40 *per Annum*, out of the whole Maſs of Mankind) which Quotient

is 12570 Millions; whereunto may be added for thoſe that died before the Flood, enough to make the laſt-mentioned Number 20 Thouſand Millions, as the full Number of all that died, from the Beginning of the World, to the Year 1682; unto which, if 320 Millions, the Number of thoſe who are now alive, be added, the Total of the Quick and the Dead, will amount but unto one fifth Part of the Graves, which the Surface of *Ireland* will afford, without ever putting two Bodies into any one Grave; for there be in *Ireland* 28 Thouſand ſquare *Engliſh* Miles, each whereof will afford about 4 Millions of Graves, and conſequently above 114 Thouſand Millions of Graves, *viz.* about five Times the Number of the Quick and the Dead, which ſhould ariſe at the laſt Day, in caſe the ſame had been in the Year 1682.

3. Now, if there may be Place for five Times as many Graves in *Ireland*, as are ſufficient for all that ever died; and if the Earth of one Grave weigh five Times as much as the Body interr'd therein, then a Turf, leſs than a Foot thick, pared off from a fifth Part of the Surface of *Ireland*, will be equivalent in Bulk and Weight to all the Bodies that ever were buried; and may ſerve as well for that Purpoſe, as the two Mountains afore-mentioned in the Body of this Diſcourſe. From all which it is plain, how madly they were miſtaken, who did ſo petulantly vilify what the Holy Scriptures have delivered.

OBSER-

OBSERVATIONS

UPON THE

DUBLIN-Bills of Mortality, 1681.

AND THE

State of that CITY.

THE Obſervations upon the *London* Bills of Mortality have been a new Light to the World; and the like Obſervation upon thoſe of *Dublin*, may ſerve as Snuffers to make the ſame Candle burn clearer.

The *London* Obſervations flowed from Bills regularly kept for near One hundred Years; but theſe are ſqueezed out of ſix ſtragling *London* Bills, out of Fifteen *Dublin* Bills, and from a Note of the Families and Hearths in each Pariſh of *Dublin*; which are all digeſted into the one Table or Sheet annexed, conſiſting of Three Parts, marked A, B, C; being indeed the A, B, C, of publick Oeconomy, and even of that Policy which tends to Peace and Plenty.

Obſervations upon the Table A.

1. THE total of the Burials in *London*, (for the ſaid ſix ſtragling Years mentioned in the Table A) is 120170; whereof the Medium or ſixth Part is 20028; and exceeds the Burials of *Paris*, as may appear by the late Bills of that City.

2. The

2. The Births, for the ſame Time, are 73683, the Medium or ſixth Part whereof is 12280, which is about five eighth Parts of the Burials; and ſhews, that *London* would in Time decreaſe quite away, were it not ſupplied out of the Country, where are about five Births for four Burials, the Proportion of Breeders in the Country being greater than in the City.

3. The Burials in *Dublin* for the ſaid ſix Years, were 9865, the ſixth Part or Medium whereof is, 1644, which is about the twelfth Part of the *London* Burials; and about a fifth Part over. So as the People of *London* do hereby ſeem to be above twelve Times as many as thoſe of *Dublin*.

4. The Births in the ſame Time at *Dublin*, are 6157, the ſixth Part or Medium whereof is 1026, which is alſo about five eighth Parts of the 1644 Burials; which ſhews, that the Proportion between Burials and Births are alike at *London* and *Dublin*, and that the Accompts are kept alike; and conſequently are likely to be true, there being no Confederacy for that Purpoſe: Which if they be true, we then ſay,

5. That the Births are the beſt Way (till the Accompts of the People ſhall be purpoſely taken) whereby to judge of the Increaſe and Decreaſe of People, that of Burials being ſubject to more Contingencies and Variety of Cauſes.

6. If Births be as yet the Meaſure of the People, and that the Births (as has been ſhewn) are as five to eight, then eight fifths of the Births is the Number of the Burials, where the Year was

not conſiderable for extraordinary Sickneſs or Salubrity; and is the Rule whereby to meaſure the ſame. As for Example: The Medium of Births in *Dublin* was 1026, the eight fifths whereof is 1641, but the real Burials were 1644; ſo as in the ſaid Years they differed little from the 1641, which was the Standard of Health: and conſequently, the Years 1680, 1674, and 1668, were ſickly Years, more or leſs, as they exceeded the ſaid Number 1641; and the reſt were healthful Years, more or leſs, as they fell ſhort of the ſame Number. But the City was more or leſs populous, as the Births differed from the Number 1026; *viz.* populous in the Years 1680, 1679, 1678, and 1668: For other Cauſes of this Difference in Births are very occult and uncertain.

7. What hath been ſaid of *Dublin*, ſerves alſo for *London*.

8. It hath already been obſerv'd by the *London* Bills, that there are more Males than Females. It is to be further noted, that in theſe ſix *London* Bills alſo, there is not one Inſtance either in the Births or Burials to the contrary.

9. It hath been formerly obſerv'd, that in the Years wherein moſt dye, feweſt are born, and *vice verſâ*. The ſame may be further obſerved in Males and Females, *viz.* When feweſt Males are born, then moſt dye: for here the Males dyed as Twelve to Eleven, which is above the mean Proportion of Fourteen to Thirteen; but were born but as Nineteen to Eighteen, which is below the ſame.

Ob-

Obſervations upon the Table B.

1. FROM the Table B, it appears, that the Medium of the fifteen Years Burials, (being 24199) is 1613, whereas the Medium of the other ſix Years in the Table A, was 1644, and that the Medium of the fifteen Years Births (being in all 14795) is 984, whereas the Medium of the ſaid other ſix Years, was 1026. That is to ſay, there were both fewer Births and Burials in theſe fifteen Years, than in the other ſix Years; which is a probable Sign that at a Medium there were fewer People alſo.

2. The Medium of Births for the fifteen Years being 984, whereof eight Fifths (being 1576) is the Standard of Health for the ſaid fifteen Years; and the Triple of the ſaid 1576, being 4728, is the Standard for each of the Ternaries of the fifteen Years within the ſaid Table.

3. That 2952, the Triple of 984 Births, is for each Ternary the Standard of Peoples Increaſe and Decreaſe from the Year 1666 to 1680 incluſive, *viz.* The People increaſed in the ſecond Ternary, and decreaſed from the ſame in the third and fourth Ternaries, but re-increaſed in the fifth Ternary beyond any other.

4. That the laſt Ternary was withal very healthful, the Burials being but 4624, *viz.* below 4728, the Standard.

5. That according to this Proportion of Increaſe, the Houſes of *Dublin* have probably increaſed alſo.

Ob-

Obſervations upon the Table C.

1. FIRST, from the Table C, it appears, 1. That the Houſing of *Dublin* is ſuch, as that there are not five Hearths in each Houſe one with another, but nearer five than four.

2. That in St. *Warburgh*'s Pariſh are near ſix Hearths to an Houſe. In St. *John*'s five. In St. *Michael*'s above five. In St. *Nicholas within* above ſix. In *Chriſt-Church* above ſeven. In St. *James*'s, and St. *Catherine*'s, and in St. *Michan*'s, not four. In St. *Kevan*'s about four.

3. That in St. *James*'s, St. *Michan*'s, St. *Bride*'s, St. *Warburgh*, St. *Andrew*'s, St. *Michael*'s, and St. *Patrick*'s, all the Chriſtenings were but 550, and the Burials 1055, *viz.* near double; and that in the reſt of the Pariſhes the Chriſtenings were five, and the Burials ſeven, *viz.* as 457 to 634. Now whether the Cauſe of this Difference were Negligence in Accompts, or the Greaterneſs of the Families, *&c.* is worth inquiring.

4. It is hard to ſay in what Order (as to Greatneſs) theſe Pariſhes ought to ſtand, ſome having moſt Families; ſome moſt Hearths, ſome moſt Births, and others moſt Burials. Some Pariſhes exceeding the reſt in two, others in three of the ſaid four Particulars, but none in all four. Wherefore this Table ranketh them according to the Plurality of the ſaid four Particulars wherein each excelleth the other.

5. The *London* Obſervations reckon eight Heads to be in each Family; according to which Eſtimation, there are 32000 Souls in the 4000 Fami-

Families of *Dublin*; which is but half of what moſt Men imagine; of which but about one ſixth Part are able to bear Arms, beſides the Royal Regiment.

6. Without the Knowledge of the true Number of People, as a Principle, the whole Scope and Uſe of the keeping Bills of Births and Burials is impaired; wherefore by laborious Conjectures and Calculations to deduce the Number of People from the Births and Burials, may be ingenious, but very prepoſterous.

7. If the Number of Families in *Dublin* be about 4000, then ten Men, in one Week (at the Charge of about five Pound, ſurveying eight Families in an Hour) may directly, and without Algebra, make an Accompt of the whole People, expreſſing their ſeveral Ages, Sex, Marriages, Title, Trade, Religion, *&c.* and thoſe who ſurvey the Hearths, or the Conſtables or Pariſh-Clerks, (may, if required) do the ſame *ex Officio*, and without other Charge, by the Command of the chief Governor, the Dioceſan, or the Mayor.

8. The Bills of *London* have ſince their Beginning, admitted ſeveral Alterations and Improvements; and eight or ten Pound *per Annum* Surcharge, would make the Bills of *Dublin* to exceed all others, and become an excellent Inſtrument of Government. To which Purpoſe the Forms for Weekly, Quarterly, and Yearly Bills are humbly recommended, *viz.*

Yearly

Anno.	Yearly Bills of Mortality for LONDON and DUBLIN.				LONDON.			
					Burials.		Births.	
	Burials.	Births.	Burials.	Births.	Males	Females	Males.	Females.
1680	21053	12747	1826	1096	11039	10044	6543	6199
1679	21730	12288	1397	1061	11154	10576	6247	6041
1678	20678	12601	1401	1045	10681	9977	6568	6033
1674	21201	11851	2106	942	11000	10196	6113	5738
1672	18230	12563	1436	987	9560	8060	6443	6120
1668	17278	11633	1699	1026	9111	8167	6073	5566
	120170	73683	9865	6157	62545	57030	37992	35697
The medium or 6th part whereof is	20028	12280	1644	1026	10424	9505	6332	5940

A

DUBLIN.				
Anno.	Burials.	Births.	In Ternaries of Years.	
1666	1480	952		
1667	1642	1001	4821	2979
1668	1699	1026		
1669	1666	1000		
1670	1713	1067	5353	3070
1671	1974	1003		
1672	1436	967		
1673	1531	933	5073	2842
1674	2106	942		
1675	1578	823		
1676	1391	952	4328	2672
1677	1359	897		
1678	1401	1045		
1679	1397	1061	4624	3202
1680	1826	1096		
	24199	14765	24199	14765
The medium or 15th part whereof is	1613	984	1613	984

B

The

	The Parishes of *DUBLIN.*	*Anno* 1671.		*An.* 1670, 71, & 72. at a Medium.	
		Families.	Hearths.	Burials.	Births.
1	St. *Katherine*'s and St. *James*,	661	2399	161	290
2	St. *Nicholas* without,	490	2348	207	262
3	St. *Michan*'s,	656	2301	127	221
4	St. *Andrew*'s with *Donabrook*,	483	2123	108	178
5	St. *Bridget*'s,	416	1989	70	100
6	St. *John*'s,	244	1337	70	138
7	St. *Warbrough*,	267	1650	54	108
8	St. *Audaens*,	216	1081	53	121
9	St. *Michael*,	140	793	44	59
10	St. *Keavens*,	106	433	64	133
11	St. *Nicholas* within,	93	614	28	34
12	St. *Patrick*'s Liberties,	52	255	21	44
13	Christ-Church and *Trinity*-College *per* Estimate,	26	197		1
	Houses built between 1671, and 1681, *per* Estimate,	3850 150	17520 550	1007	1689
		4000	18070		

A Weekly

C D

A Weekly Bill of Mortality for the City of *Dublin*,

Ending the Day of 1681.

PARISHES Names.	Births.	Males.	Females.	Burials.	Under 16 years old.	Above 16 years old.	Plague.	Small Pox	Measles.	Spotted Fever.
1 St. *Katherines* and St. *James*,										
2 St. *Nicholas* without,										
3 St. *Michans*,										
4 St. *Andrews* with *Donabrook*,										
5 St. *Bridgets*,										
6 St. *Johns*,										
7 St. *Warbrough*,										
8 St. *Audoens*,										
9 St. *Michael*,										
10 St. *Kevins*,										
11 St. *Nicholas* within,										
12 St. *Patrick's* Liberties,										
13 Christ-Church and *Trinity*-College,										
Totals,										

A Quar-

A Quarterly Bill of Mortality,

Beginning and ending for the City of *Dublin*.

PARISHES Names.	Births. 1.	Marriages. 2.	Buried of Under 16 years old.	Buried of Above 60 years old.	Plague, Small Pox, Spotted Fever, Measles.	Dropsie, Consumption.	Stone, Gout, Fever.	Quinsey, Plurisie, Sudden Death.	Aged above 70. years old.	Infants under 2. years old.	All other Casualties.
1 St. *Katherines* and St. *James*,											
2 St. *Nicholas* without,											
3 St. *Michans*,											
4 St. *Andrews* with *Donabrook*,											
5 St. *Bridgets*,											
6 St. *Johns*,											
7 St. *Warbrough*,											
8 St. *Audaens*,											
9 St. *Michael*,											
10 St. *Keavens*,											
11 St. *Nicholas* within,											
12 St. *Patrick's* Liberties,											
13 Christ-Church and *Trinity*-College.											
Totals,											

An Account of the People of *Dublin* for one Year,

Ending the 24th of *March*, 1681-2.

PARISHES Names.	Number of Persons.	Whereof Males.	Whereof Females.	Married Persons.	Persons of Under 16 Years old.	Persons of Above 60 Years old.	Of all other Religions.	Papists of above 16 Years old.	Protestants of above 16 Years old.	Births.	Burials.	Marriages.
1 St. *Katherines* and St. *James*,												
2 St. *Nicholas* without,												
3 St. *Michans*,												
4 St. *Andrews* with *Donabrook*,												
5 St. *Bridgets*,												
6 St. *Johns*,												
7 St. *Warbrough*,												
8 St. *Audoens*,												
9 St. *Michael*,												
10 St. *Keavins*,												
11 St. *Nicholas* within,												
12 St. *Patrick's* Liberties,												
13 Christ-Church and *Trinity*-College.												
Totals,												

Casu-

Caſualties and Diſeaſes.

Aged above 70 Years.		Head-ach, and Megrim.	
Abortive and Still-born.		Epilepſie, and Planet.	
Childbed-women.		Fever, and Ague.	
Convulſion.		Pleuriſie.	
Teeth.		Quinſey.	
Worms.		Executed. Murder'd, Drown'd.	
Gout, and Sciatica.		Plague, and Spotted-Fever.	
Stone.		Griping of the Guts.	
Palſey.		Scowring, Vomiting, Bleeding.	
Conſumption and French Pox.		Small-Pox.	
Dropſie, and Tympany.		Meaſles.	
Rickets, and Livergrown.		Neither of all the other Sorts.	

A

POSTSCRIPT

TO THE

STATIONER.

WHEREAS you complain, that theſe Obſervations make no ſufficient Bulk, I could anſwer you, that I wiſh the Bulk of all Books were leſs; but do nevertheleſs comply with you in adding what follows, *viz.*

1. That the Pariſhes of *Dublin* are very unequal; ſome having in them above ſix Hundred Families, and others under Thirty.

2. That thirteen Pariſhes are too few for four Thouſand Families; the middling Pariſhes of *London* containing One hundred and twenty Families; according to which Rate, there ſhould be about Thirty three Pariſhes in *Dublin*.

3. It is ſaid, that there are eighty four Thouſand Houſes or Families in *London*, which is twenty-one Times more than are in *Dublin*; and yet the Births and Burials of *London* are but twelve Times thoſe of *Dublin*: which ſhews, that the Inhabitants of *Dublin* are more crowded and ſtreightned in their Houſing, than

thoſe

thoſe of *London*; and conſequently, that to increaſe the Buildings of *Dublin*, will make that City more conformable to *London*.

4. I ſhall alſo add ſome Reaſons for altering the preſent Forms of the *Dublin* Bills of Mortality, according to what hath been here recommended, *viz.*

1. We give the Diſtinctions of Males and Females in the Births only; for that the Burials muſt, at one Time or another, be in the ſame Proportion with the Births.

2. We do in the Weekly and Quarterly Bills propoſe, that Notice be taken in the Burials of what Numbers dye above Sixty and Seventy, and what under Sixteen, Six, and two Years old; foreſeeing good Uſes to be made of that Diſtinction.

3. We do in the yearly Bill, reduce the Caſualties to about Twenty-four, being ſuch as may be diſcerned by common Senſe and without Art; conceiving that more will but perplex and imbroil the Account. And in the Quarterly Bills, we reduce the Diſeaſes to three Heads, *viz.* contagious, acute, and chronical; applying this Diſtinction to Pariſhes, in Order to know how the different Situation, Soil, and Way of living in each Pariſh, doth diſpoſe Men to each of the ſaid three Species: and in the Weekly Bills we take Notice not only of the Plague, but of the other contagious Diſeaſes in each Pariſh; that Strangers and fearful Perſons may thereby know how to diſpoſe of themſelves.

4. We

4. We mention the Number of the People, as the fundamental Term in all our Proportions; and without which, all the reſt will be almoſt fruitleſs.

5. We mention the Number of Marriages made in every Quarter, and in every Year; as alſo the Proportion which married Perſons bear to the whole; expecting in ſuch Obſervations to read the Improvement of the Nation.

6. As for Religions, we reduce them to three, *viz.* 1. Thoſe who have the Pope of *Rome* for their Head. 2. Who are governed by the Laws of their Country. 3. Thoſe who rely reſpectively upon their own private Judgments. Now whether theſe Diſtinctions ſhould be taken Notice of or not, we do but faintly recommend, ſeeing many Reaſons *pro* and *con* for the ſame: and therefore although we have mentioned it as a Matter fit to be conſidered, yet we humbly leave it to Authority.

Further

DUBLIN, 1682.				
Pariſhes.	Houſes.	Fire-places.	Baptiz'd.	Buried.
St. *James*	272	836	122	306
St. *Katherines*	540	2198		
St. *Nicholas* without, and St. *Patricks*	1064	4082	145	414
St. *Bridgets*	395	1903	68	149
St. *Audaen*	276	1510	56	164
St. *Michael*	174	884	34	50
St. *Johns*	302	1636	74	101
St. *Nicholas* within, and Chriſt-Church *Lib.*	153	902	26	52
St. *Warbrough*	240	1638	45	105
St. *Michans*	938	3516	124	389
St. *Andrews*	864	3638	131	300
St. *Kevans*	554	2120	87	233
Donobrook	253	506		
	6025	25369	912	2263

Further Obſervations upon the Dublin *Accompts of Baptiſms and Burials, Houſes and Hearths.*

THE Table hath been made for the Year 1682, wherein is to be noted,

1. That the Houſes which *Anno* 1671, were but 3850 are *Anno* 1682, 6025; but whether this Difference is cauſed by the real Encreaſe of Houſing, or by Fraud and Defect in the former Accompts, is left to Conſideration. For the Burials or People have increaſed but from 1696, to 2263, according to which Proportion, the 3850 Houſes *Anno* 1671, ſhould *Anno* 1682 have been but 5143, wherefore ſome Fault may be

be ſuſpected as aforeſaid, when farming the Hearth-money was in Agitation.

2. The Hearths have encreaſed according to the Burials, and $\frac{1}{3}$ of the ſaid Increaſe more, *viz.* The Burials *Anno* 1671 were 1696, the $\frac{1}{3}$ whereof is 563, which put together makes 2259, which is near the Number of Burials *Anno* 1682. But the Hearths *Anno* 1671, were 17500, whereof the $\frac{1}{3}$ is 5833, making in all but 23333; whereas the whole Hearths *Anno* 1682 were 25369, *viz.* $\frac{1}{3}$ and better of the ſaid 5833 more.

3. The Houſes were *Anno* 1671, but 3850, which if they had increaſed *Anno* 1682 but according to the Burials, they had been but 5143, or according to the Hearths, had been but 5488, whereas they appear 6025, increaſing double to the Hearths. So as 'tis likely there hath been ſome Error in the ſaid Account of the Houſes, unleſs the new Houſes be very ſmall, and have but one Chimney a piece, and that $\frac{1}{4}$ Part of them are untenanted. On the other Hand, 'tis more likely that when 1696 died *per An.* there were near 6000; for 6000 Houſes at 8 Inhabitants *per* Houſe, would make the Number of the People to be 48 Thouſand, and the Number of 1696 that died according to the Rule of 1 out of 30, would have made the Number of Inhabitants about 50 Thouſand: For which Reaſon I continue to believe there was ſome Error in the Accompt of 3850 Houſes as aforeſaid, and the rather becauſe there is no Ground from Expe-

Experience to think that in 11 Years, the Houſes in *Dublin* have increaſed from 3850 to 6025.

Moreover, I rather think that the Number of 6025 is yet ſhort, becauſe that Number at 8 Heads *per* Houſe makes the Inhabitants to be but 48200; whereas the 2263 who died in the Year 1682, according to the aforemention'd Rule of one dying out of 30 makes the Number of People to be 67890; the Medium betwixt which Number and 48200 is 58045, which is the beſt Eſtimate I can make of that Matter, which I hope Authority will ere long rectify, by direct and exact Enquiries.

4. As to the Births, we ſay that *Anno* 1640, 1641, and 1642, at *London*, juſt before the Troubles in Religion began, the Births were $\frac{1}{6}$ of the Burials, by Reaſon I ſuppoſe of the Greaterneſs of Families in *London* above the Country, and the fewer Breeders, and not for Want of regiſtering. Wherefore deducting $\frac{1}{6}$ of 2263, which is 377, there remains 1886 for the probable Number of the Births in *Dublin* for the Year 1682; whereas but 912 are repreſented to have been chriſten'd in that Year, though 1023 were chriſten'd *Anno* 1671, when there died but 1696; which Decreaſing of the Chriſtenings, and Increaſing of the Burials, ſhews the Increaſe of Non-regiſtering in the legal Books, which muſt be the Increaſe of *Roman Catholicks* at *Dublin*.

The

The Scope of this whole Paper therefore is, That the People of *Dublin* are rather 58000, than 32000; and that the Diſſenters, who do not regiſter their Baptiſms, have increaſed from 391 to 974: but of Diſſenters, none have increaſed but the *Roman Catholicks*, whoſe Numbers have increaſed from about 2 to 5 in the ſaid Years. The exacter Knowledge whereof, may alſo be better had from direct Enquiries.

TWO

TWO

ESSAYS

IN

POLITICAL ARITHMETICK,

Concerning

The People, Houſing, Hoſpitals, &c.

OF

LONDON and PARIS.

By Sir WILLIAM PETTY, *Fellow of the* Royal Society.

——— *Qui ſciret Regibus uti*
Faſtidiret olus ———

TO THE

KING'S

Moſt Excellent MAJESTY.

I Do preſume, in a very ſmall Paper, to ſhew your Majeſty, that your City of *London* ſeems more conſiderable than the two beſt Cities of the *French* Monarchy, and for ought I can find, greater than any other of the Univerſe; which becauſe I can ſay without Flattery, and by ſuch Demonſtration as your Majeſty can examine, I humbly pray your Majeſty to accept from

Your MAJESTY's

Moſt Humble, Loyal,

and Obedient Subject,

WILLIAM PETTY.

AN

ESSAY

IN

POLITICAL ARITHMETICK,

By Sir *WILLIAM PETTY*,

Tending to prove that London *hath more People and Houſes than the Cities of* Paris *and* Rouen *put together, and is alſo more conſiderable in ſeveral other Reſpects.*

1. THE Medium of the Burials at *London* in the three laſt Years, *viz.* 1683, 1684 and 1685, (wherein there was no extraordinary Sickneſs, and wherein the Chriſtenings do correſpond in their ordinary Proportions with the Burials and Chriſtenings of each Year one with another) was 22337, and the like Medium of Burials for the three laſt *Paris* Bills we could procure, *viz.* for the Years 1682, 1683, and 1684 (whereof the laſt as appears by the Chriſtenings to have been very ſickly) is 19887.

2. The City of *Briſtol* in *England* appears to be by good Eſtimate of its Trade and Cuſtoms, as

as great as *Rouen* in *France*, and the City of *Dublin* in *Ireland* appears to have more Chimnies than *Briſtol*, and conſequently more People, and the Burials in *Dublin* were *Anno* 1682 (being a ſickly Year) but 2263.

3, Now the Burials of *Paris* (being 19887) being added to the Burials of *Dublin* (ſuppoſed more than at *Rouen*) being 2263, makes but 22150, whereas the Burials of *London* were 187 more, or 22337, or as about 6 to 7.

4. If thoſe who die unneceſſarily, and by Miſcarriage in *L' Hoſtel Dieu* in *Paris* (being above 3000) as hath been elſewhere ſhewn, or any Part thereof, ſhould be ſubtracted out of the *Paris* Burials afore-mentioned, then our Aſſertion will be ſtronger, and more proportionable to what follows concerning the Houſes of thoſe Cities, *viz.*

5. There were burnt at *London*, *Anno* 1666, above 13000 Houſes, which being but a fifth Part of the whole, the whole Number of Houſes in the ſaid Year, were above 65000; and whereas the ordinary Burials of *London* have increaſed between the Years 1666 and 1686, above one third, the Total of the Houſes at *London Anno* 1686, muſt be about 87000, which *Anno* 1682, appeared by Accompt to have been 84000.

6. *Monſieur Morery*, the great *French* Author of the late *Geographical Dictionaries*, who makes *Paris* the greateſt City in the World, doth reckon but 50000 Houſes in the ſame, and other Authors and knowing Men much leſs; nor are there

there full 7000 Houſes in the City of *Dublin*; ſo as if the 50000 Houſes of *Paris* and the 7000 Houſes in the City of *Dublin* were added together, the Total is but 57000 Houſes; whereas thoſe of *London* are 87000 as aforeſaid, or as 6 to 9.

7. As for the Shipping and foreign Commerce of *London*, the common Senſe of all Men doth judge it to be far greater than that of *Paris* and *Rouen* put together.

8. As to the Wealth and Gain accruing to the Inhabitants of *London* and *Paris* by Law-ſuits (or *La Chicane*) I only ſay that the Courts of *London* extend to all *England* and *Wales*, and affect ſeven Millions of People, whereas thoſe of *Paris* do not extend near ſo far: Moreover there is no palpable conſpicuous Argument at *Paris* for the Number and Wealth of Lawyers like the Buildings and Chambers in the two *Temples*, *Lincoln's Inn*, *Gray's Inn*, *Doctors Commons*, and the ſeven other Inns, in which are Chimnies, which are to be ſeen at *London*, beſides many Lodgings, Halls, and Offices relating to the ſame.

9. As to the plentiful and eaſy living of the People we ſay,

1. That the People of *Paris* to thoſe of *London*, being as about 6 to 7, and the Houſes of the ſame as about 6 to 9, we infer that the People do not live at *London* ſo cloſe and crouded as at *Paris*, but can afford themſelves more Room and Liberty.

2. That at *London* the Hoſpitals are better and more deſirable than thoſe of *Paris*, for that in the beſt at *Paris* there die 2 out of 15, whereas at *London* there die out of the worſt ſcarce 2 of 16, and yet but a fiftieth Part of the whole die out of the Hoſpitals at *London*, and $\frac{2}{5}$ or 20 Times that Proportion die out of the *Paris* Hoſpitals which are of the ſame Kind; that is to ſay, the Number of thoſe at *London* who chuſe to lie ſick in Hoſpitals rather than in their own Houſes, are to the like People of *Paris* as one to twenty; which ſhews the greater Poverty or Want of Means in the People of *Paris* than thoſe of *London*.

3. We infer from the Premiſes, *viz.* the dying ſcarce 2 of 16 out of the *London* Hoſpitals, and about 2 of 15 in the beſt of *Paris*, (to ſay nothing of *L' Hoſtel Dieu*) that either the Phyſicians and Chirurgeons of *London* are better than thoſe of *Paris*, or that the Air of *London* is more wholeſome.

10. As for the other great Cities of the World, if *Paris* were the greateſt, we need ſay no more in Behalf of *London*. As for *Pequin* in *China*, we have no Account fit to reaſon upon; nor is there any thing in the Deſcription of the two late Voyages of the *Chineſe* Emperor from that City into Eaſt and Weſt *Tartary*, in the Years 1682 and 1683, which can make us recant what we have ſaid concerning *London*. As for *Dely* and *Agra* belonging to the Mogul, we find nothing againſt our Poſition, but much to

ſhew

ſhew the vaſt Numbers which attend that Emperor in his Buſineſs and Pleaſures.

11. We ſhall conclude with *Conſtantinople* and *Gran Cairo*; as for *Conſtantinople* it hath been ſaid by one who endeavour'd to ſhew the Greatneſs of that City, and the Greatneſs of the Plague which reigned in it, that there died 1500 *per Diem*, without other Circumſtances: To which we anſwer, that in the Year 1665 there died in *London* 1200 *per Diem*, and it hath been well proved that the Plague of *London* never carried away above $\frac{1}{5}$ of the People, whereas it is commonly believed that in *Conſtantinople*, and other Eaſtern Cities, and even in *Italy* and *Spain*, that the Plague takes away $\frac{2}{5}$ one half or more; wherefore where 1200 is but $\frac{1}{5}$ of the People, it is probable that the Number was greater, than where 1500 was $\frac{2}{5}$ or one half, *&c.*

12. As for *Gran Cairo* it is reported, that 73000 died in 10 Weeks or 1000 *per Diem*, where note, that at *Gran Cairo* the Plague comes and goes away ſuddenly, and that the Plague takes away 2 or $\frac{3}{5}$ Parts of the People as aforeſaid; ſo as 73000 was probably the Number of thoſe that died of the Plague in one whole Year at *Gran Cairo*, whereas at *London Anno* 1665, 97000 were brought to Account to have died in that Year. Wherefore it is certain that that City wherein 97000 was but $\frac{1}{5}$ of the People, the Number was greater than where 73000 was $\frac{2}{5}$ or the half.

We therefore conclude, that *London* hath more People, Houſes, Shipping, and Wealth, than *Paris* and *Rouen* put together; and for ought yet appears, is more conſiderable than any other City in the Univerſe, which was propounded to be proved.

AN

AN ESSAY IN POLITICAL ARITHMETICK,

By Sir *WILLIAM PETTY*,

Tending to prove that in the Hoſpital called L'Hoſtel Dieu *at* Paris, *there die above* 3000 per Annum, *by Reaſon of ill Accommodation.*

1. IT appears that *Anno* 1678 there entered into the Hoſpital of *La Charité* 2647 Souls, of which there died there within the ſaid Year 338, which is above an eighth Part of the ſaid 2647, and that in the ſame Year there entered into *L'Hoſtel Dieu* 21491, and that there died out of that Number 5630, which is above one Quarter, ſo as about half the ſaid 5630, being 2815, ſeem to have died for Want of as good Uſage and Accommodation as might have been had at *La Charité*.

2. Moreover in the Year 1679 there entered into *La Charité* 3118, of which there died 452, which is above a ſeventh Part, and in the ſame year there entered into *L' Hoſtel Dieu* 28635, of which there died 8397, and in both the ſaid

Loſs of the Subjects of *France* in that Hoſpital ſeems to be 60 Times 3506 Livres *Sterl. per Annum, viz.* 210 Thouſand 360 Livres *Sterl.* equivalent to about two Millions 524 Thouſand 320 *French* Livres.

7. It hath appeared that there came into *L' Hoſtel Dieu* at a Medium 25063 *per Annum*, or 2089 *per Menſem*, and that the whole Stock of what remained in the precedent Months is at a Medium about 2108 (as may appear by the third Line of the Table N° 5, which ſhall be ſhortly publiſhed) *viz.* the Medium of Months is 2410 for the ſickly Year 1679, whereunto 1806 being added, as the Medium of Months for the Year 1678, makes 4216, the Medium whereof is the 2108 above-mentioned; which Number being added to the 2089 which entered each Month, makes 4197 for the Number of Sick which are ſuppoſed to be always in *L' Hoſtel Dieu* one Time with another.

8. Now if 60 *French* Livres *per Annum* for each of the ſaid 4197 ſick Perſons were added to the preſent ordinary Expence of that Hoſpital (amounting to an Addition of 251 Thouſand 820 Livres) it ſeems that ſo many Lives might be ſaved as are worth above ten Times that Sum, and this by doing a manifeſt Deed of Charity to Mankind.

Memorandum, *That* Anno 1685, *the Burials of* London *were* 23222, *and thoſe of* Amſterdam 6245; *from whence, and the Difference of Air, 'tis probable that the People of* London *are quadruple to thoſe of* Amſterdam.

OB-

OBSERVATIONS UPON THE CITIES OF LONDON and ROME.

1. THAT before the Year 1630, the Chriſtenings at *London* exceeded the Burials of the ſame, but about the Year 1655 they were ſcarce half; and now about two Thirds.

2. Before the Reſtauration of Monarchy in *England*, *Anno* 1660, the People of *Paris* were more than thoſe of *London* and *Dublin* put together, whereas now, the People of *London* are more than thoſe of *Paris* and *Rome*, or of *Paris* and *Rouen*.

3. *Anno* 1665 one fifth Part of the then People of *London* or 97 Thouſand died of the Plague, and in the next Year 1666, 13 Thouſand Houſes, or one fifth Part of all the Houſing of *London* were burnt alſo.

4. At the Birth of *Chriſt*, old *Rome* was the greateſt City of the World, and *London* the greateſt

greateſt at the Coronation of King *James* the Second, and near 6 Times as great as the preſent *Rome*, wherein are 119 Thouſand Souls beſides *Jews*.

5. In the Years of King *Charles* the Second his Death and King *James* the Second his Coronation (which were neither of them remarkable for extraordinary Sicklineſs or Healthfulneſs) the Burials did wonderfully agree, *viz.* *Anno* 1684, they were 23202, and *Anno* 1685 they were 23222, the Medium whereof is 23212. And the Chriſtenings did very wonderfully agree alſo, having been *Anno* 1684, 14702, and *Anno* 1685, 14732, the Medium whereof is 14717, which Conſiſtence was never ſeen before, the ſaid Number of 23212 Burials making the People of *London* to be 696360, at the Rate of one dying *per Annum* out of 30.

6. Since the great Fire of *London*, *An.* 1666 about 7 Parts of 15 of the preſent vaſt City hath been new built, and is with its People increaſed near one half, and become equal to *Paris* and *Rome* put together, the one being the Seat of the great *French* Monarchy, and the orher of the Papacy.

FIVE

FIVE ESSAYS

IN

POLITICAL ARITHMETICK,

VIZ.

I. Objections from the City of *Rey* in *Perſia*, and from Monſieur *Auzout*, againſt two former Eſſays, anſwered, and that *London* hath as many People as *Paris*, *Rome*, and *Rouen* put together.

II. A Compariſon between *London* and *Paris* in 14 Particulars.

III. Proofs that at *London*, within its 134 Pariſhes named in the Bills of Mortality, there live about 696 Thouſand People.

IV. An Eſtimate of the People in *London*, *Paris*, *Amſterdam*, *Venice*, *Rome*, *Dublin*, *Briſtol*, and *Rouen*, with ſeveral Obſervations upon the ſame.

V. Concerning *Holland*, and the reſt of the Seven *United Provinces*.

By Sir WILLIAM PETTY, *Fellow of the* Royal Society.

Invidiam augendo ulciſcar.

London 1687.

TO THE

KING'S

Moſt Excellent MAJESTY.

SIR,

YOUR Majeſty having graciouſly accepted my two late Eſſays about the Cities and Hoſpitals of *London* and *Paris*, as alſo my Obſervations on *Rome* and *Rouen*; I do (after ſix Months waiting for what may be ſaid againſt my ſeveral Doctrines, by the able Men of *Europe*) humbly preſent your Majeſty with a few other Papers upon the ſame Subject, to ſtrengthen, explain and enlarge the former; hoping by ſuch real Arguments, better to praiſe and magnify your Majeſty, than by any other the moſt ſpecious Words and Elogies that can be imagined by

Your MAJESTY'S
Moſt Humble, Loyal,
and Obedient Subject,

1687, WILLIAM PETTY.

The FIRST ESSAY.

IT could not be expected that an Aſſertion of *London*'s being bigger than *Paris* and *Rouen*, or than *Paris* and *Rome* put together, and bigger than any City of the World, ſhould eſcape uncontradicted; and 'tis alſo expected, that I (if continuing in the ſame Perſuaſion) ſhould make ſome Reply to thoſe Contradictions. In order whereunto,

I begin with the ingenious Author of the *Republique des Lettres*, who ſaith that *Rey* in *Perſia* is far bigger than *London*, for that in the 6th Century of Chriſtianity (I ſuppoſe *An.* 550, the Middle of that Century) it had 15000, or rather 44000 Moſchees, or *Mahometan* Temples; to which I reply, that I hope this Objector is but in Jeſt, for that *Mahomet* was not born till about the Year 570, and had no Moſchees till about 50 Years after.

In the next Place I reply to the excellent Monſieur *Auzout*'s Letters from *Rome*, who is content that *London*, *Weſtminſter* and *Southwark*, may have as many People as *Paris* and its Suburbs; and but faintly denieth, that all the Houſing within the Bills, may have almoſt as many People as *Paris* and *Rouen*, but ſaith that ſeveral Pariſhes inſerted into theſe Bills, are diſtant from, and not contiguous with *London*, and that *Grant* ſo underſtood it.

To

To which (as his main if not his only Objection) we anſwer: 1. That the *London* Bills appear in *Grant*'s Book, to have been always ſince the Year 1636, as they now are. 2. That about 50 Years ſince, three or 4 Pariſhes, formerly ſomewhat diſtant, were joined by interpoſed Buildings, to the Bulk of the City, and therefore then inſerted into the Bills. 3. That ſince 50 Years, the whole Buildings being more than double, have perfected that Union, ſo as there is no Houſe within the ſaid Bills, from which one may not call to ſome other Houſe. 4. All this is confirmed by Authority of the King and City, and the Cuſtom of 50 Years. 5. That there are but 3 Pariſhes under any Colour of this Exception, which are ſcarce $\frac{1}{50}$ Part of the Whole.

Upon the whole Matter, upon the Sight of Monſieur *Auzout*'s large Letter, dated the 19th of *November*, from *Rome*, I made Remarks upon every Paragraph thereof; but ſuppreſſing it (becauſe it look'd like a War againſt a worthy Perſon, with whom I intended none, whereas in Truth it was but a reconciling Explication of ſome Doubts) I have choſen the ſhorter and ſofter Way of anſwering Monſieur *Auzout* as followeth, *viz.*

Con-

Concerning the Number of People in London, *as alſo in* Paris, Rouen, *and* Rome, viz.

Monſieur *Auzout* alledgeth an authentick Accompt, that there are 23223 Houſes in *Paris*, wherein do live about 80 Thouſand Families, and therefore ſuppoſing 3½ Families, to live in every of the ſaid Houſes, one with another, the Number of Families will be 81280; and Monſieur *Auzout* alſo allowing 6 Heads to each Family, the utmoſt Number of People in *Paris* according to that Opinion will be	487, 680
The Medium of the *Paris* Burials was not denied by Monſieur *Auzout* to be 19887, nor that there died 3506 unneceſſarily out of *L' Hotel Dieu*; wherefore deducting the ſaid laſt Number out of the former, the neat Standard for Burials at *Paris*, will be 16381, ſo as the Number of People there, allowing but one to die out of 30 (which is more advantageous to *Paris* than Monſieur *Auzout*'s Opinion of one to die out of 25) the Number of People at *Paris* will be 491,430, more than by Monſieur *Auzout*'s own laſt mentioned Account.	491, 430

And

And the Medium of the ſaid two *Paris* Accompts is		488, 055
The Medium of the *London* Burials is really 23212, which multiplied by 30 (as hath been done for *Paris*) the Number of the People there will be		696, 360
The Number of Houſes at *London* appears by the Regiſter to be 105,315, whereunto adding $\frac{1}{10}$ Part of the ſame, or 10331, as the leaſt Number of double Families that can be ſuppoſed in *London*, the Total of Families will be 115,840; and allowing 6 Heads for each Family as was done for *Paris*, the Total of the People at *London* will be		695, 076
The Medium of the two laſt *London* Accounts is		695, 718
So as the People of *Paris* according to the above Account is	488, 055	
Of *Rouen* according to Monſieur *Auzout*'s utmoſt Demands	80, 000	693, 055
Of *Rome* according to his own Report thereof in a former Letter	125, 000	
So as there are more People at *London* than at *Paris*, *Rouen*, and *Rome*		2, 663

Memo-

Memorandum, That the Pariſhes of *Iſlington*, *Newington*, and *Hackney*, for which only there is any Colour of Non-contiguity, is not $\frac{1}{51}$ Part of what is contained in the Bills of Mortality, and conſequently *London*, without the ſaid 3 Pariſhes, hath more People than *Paris* and *Rouen* put together by } 114, 284

Which Number of 114,284 is probably more People than any other City of *France* contains.

The SECOND ESSAY.

AS for other Compariſons of *London* with *Paris*, we farther repeat and enlarge what hath been formerly ſaid upon thoſe Matters, as followeth, *viz.*

1. That 40 *per Cent.* die out of the Hoſpitals at *Paris* where ſo many die unneceſſarily, and ſcarce $\frac{1}{20}$ of that Proportion out of the Hoſpitals of *London*, which have been ſhewn to be better than the beſt of *Paris*.

2. That at *Paris* 81280 Kitchins, are within leſs than 24000 Street-doors, which makes a leſs cleanly and convenient Way of living than at *London*.

3. Where the Number of Chriſtenings are near unto, or exceed the Burials, the People are poorer, having few Servants and little Equipage.

4, The River of *Thames* is more pleaſant and navigable than the *Seyne*, and its Waters better and more wholeſome; and the Bridge of *London* is the moſt conſiderable of all *Europe*.

5. The Shipping and foreign Trade of *London* is incomparably greater than that at *Paris* and *Rouen*.

6. The Lawyers Chambers at *London* have 2772. Chimnies in them, and are worth 140 Thouſand Pounds ſterling, or 3 Millions of *French* Livres, beſides the Dwellings of their Families elſewhere.

7. The Air is more wholeſome, for that at *London* ſcarce 2 of 16 die out of the worſt Hoſpitals, but at *Paris* above 2 of 15 out of the beſt. Moreover the Burials of *Paris* are $\frac{1}{5}$ Part above and below the Medium, but at *London* not above $\frac{1}{12}$, ſo as the Intemperies of the Air at *Paris* is far greater than at *London*.

8. The Fuel cheaper, and lies in leſs Room, the Coals being an wholeſome ſulphurous Bitumen.

9. All the moſt neceſſary Sorts of Victuals, and of Fiſh, are cheaper, and Drinks of all Sorts in greater Variety and Plenty.

10. The Churches of *London* we leave to be judg'd by thinking that nothing at *Paris* is ſo great as St. *Paul*'s was, and is like to be, nor ſo beautiful as *Henry* the Seventh's Chapel.

11. On the other Hand, 'tis probable, that there is more Money in *Paris* than *London*, if the publick Revenue (groſly ſpeaking, quadruple to that of *England*) be lodged there.

12. *Paris*

12. *Paris* hath not been for theſe laſt 50 Years ſo much infeſted with the Plague as *London*; now that at *London* the Plague (which between the Year 1591 and 1666, made 5 Returns, *viz.* every 15 Years at a Medium, and at each Time carried away $\frac{1}{5}$ of the People) hath not been known for the 21 Years laſt paſt, and there is a viſible Way by God's ordinary Bleſſing to leſſen the ſame by $\frac{2}{3}$ when it next appeareth.

13. As to the Ground upon which *Paris* ſtands in reſpect of *London*, we ſay, that if there be 5 Stories or Floors of Houſing at *Paris*, for 4 at *London*, or in that Proportion, then the 82 Thouſand Families at *Paris* ſtand upon the Equivalent of 65000 *London* Houſteds, and if there be 115000 Families at *London*, and but 82000 at *Paris*, then the Proportion of the *London* Ground to that of *Paris* is as 115 to 65, or as 23 to 13.

14. Moreover *Paris* is ſaid to be an Oval of 3 *Engliſh* Miles long and $2\frac{1}{2}$ broad, the Area whereof contains but $5\frac{1}{2}$ ſquare Miles; but *London* is 7 Miles long, and $\frac{1}{14}$ broad at a Medium, which makes an Area of near 9 ſquare Miles, which Proportion of $5\frac{1}{2}$ to 9 differs little from that of 13 to 23.

15. *Memorandum*, That in *Nero*'s Time, as Monſieur *Chevreau* reporteth, there died 300 Thouſand People of the Plague in Old *Rome*; now if there died 3 of 10 then, and there, being a hotter Country, as there dies 2 of 10 at *London*, the Number of People at that Time, was but a Million, whereas at *London* they are now about

700 Thouſand. Moreover the Ground within the Walls of Old *Rome* was a Circle but of 3 Miles diameter, whoſe Area is about 7 ſquare Miles, and the Suburbs ſcarce as much more, in all about 13 ſquare Miles; whereas the built Ground at *London* is about 9 ſquare Miles as aforeſaid; which two Sorts of Proportions agree with each other, and conſequently Old *Rome* ſeems but to have been half as big again as the preſent *London*, which we offer to Antiquaries.

The THIRD ESSAY.

PROOFS that the Number of People in the 134 Pariſhes of the *London* Bills of Mortality, without Reference to other Cities, is about 696 Thouſand, *viz.*

I know but three Ways of finding the ſame.

1. By the Houſes and Families, and Heads living in each.

2. By the Number of Burials in healthful Times, and by the Proportion of thoſe that live, to thoſe that die.

3. By the Number of thoſe who die of the Plague in peſtilential Years, in Proportion to thoſe that eſcape.

The Firſt Way.

To know the Number of Houſes I uſed three Methods, *viz.*

1. The Number of Houſes which were burnt *Anno* 1666, which by authentick Report was 13200; next what Proportion the People who died out of thoſe Houſes, bore to the Whole;

Whole; which I find *Anno* 1686, to be but $\frac{1}{7}$ Part, but *Anno* 1666 to be almoſt $\frac{1}{5}$, from whence I infer the whole Houſing of *London Anno* 1666 to have been 66 Thouſand; then finding the Burials *Anno* 1666 to be to thoſe of 1686 as 3 to 4, I pitch upon 88 Thouſand to be the Number of Houſes *Anno* 1686.

2. Thoſe who have been employed in making the general Map of *London*, ſet forth in the Yeat 1682, told me that in that Year they had found above 84 Thouſand Houſes to be in *London*; wherefore *Anno* 1686, or in 4 Years more, there might be $\frac{1}{10}$ or 8400 Houſes more (*London* doubling in 40 Years) ſo as the Whole, *Anno* 1686 might be 92400.

3. I found that *Anno* 1685, there were 29325 Hearths in *Dublin*, and 6400 Houſes, and in *London* 388 Thouſand Hearths, whereby there muſt have been at that Rate 87000 Houſes in *London*. Moreover I found that in *Briſtol* there were in the ſame Year 16752 Hearths, and 5307 Houſes, and in *London* 388 Thouſand Hearths as aforeſaid; at which Rate there muſt have been 123 Thouſand Houſes in *London*, and at a Medium between *Dublin* and *Briſtol* Proportions, 105 Thouſand Houſes.

Laſtly, by Certificate from the Hearth-Office, I find the Houſes within the Bills of Mortality to be 105,315.

Having thus found the Houſes, I proceed next to the Number of Families in them; and firſt I thought that if there were 3 or 4 Families or

Kitchins in every Houſe of *Paris*, there might be 2 Families in $\frac{1}{10}$ of the Houſing of *London*; unto which Suppoſition, the common Opinion of ſeveral Friends, doth concur with my own Conjectures.

As to the Number of Heads in each Family, I ſtick to *Grant*'s Obſervation in his fifth Edition, that in Tradeſmen of *London*'s Families, there be 8 Heads one with another, in Families of higher Ranks, above 10, and in pooreſt near 5, according to which Proportions, I had upon another Occaſion pitch'd the Medium of Heads in all the Families of *England* to be $6\frac{1}{3}$, but quitting the Fraction in this Caſe, I agree with Monſieur *Auzout* for 6.

To conclude, the Houſes of *London* being 105315, and the Addition of double Families 10531 more, in all 115846; I multiplied the ſame by 6, which produced 695076 for the Number of the People.

The Second Way.

I found that the Years 1684 and 1685, being next each other, and both healthful, did wonderfully agree in their Burials, *viz.* 1684 they were 23202, and *Anno* 1685 23222, the Medium whereof is 23212; moreover that the Chriſtenings 1684 were 14,702, and thoſe *Anno* 1685 were 14730; wherefore I multiplied the Medium of Burials 23212 by 30, ſuppoſing that one dies out of 30 at *London*, which made the Number of People 696360 Souls.

Now

Now to prove that one dies out of 30 at *London*, or thereabouts, I ſay,

1. That *Grant* in his fifth Edition, affirmeth from Obſervation, that 3 died of 88 *per Annum*, which is near the ſame Proportion.

2. I found that out of healthful Places, and out of adult Perſons, there dies much fewer, as but 1 out of 50 among our Parliament Men, and that the Kings of *England* having reigned 24 Years one with another, probably lived above 30 Years each.

3, *Grant* hath ſhewn that but about 1 of 20 die *per An.* out of young Children under 10 Years old, and Monſieur *Auzout* thinks that but 1 of 40 die at *Rome*, out of the greater Proportion of adult Perſons there; wherefore we ſtill ſtick at a Medium to the Number 30.

4. In 9 Country Pariſhes lying in ſeveral Parts of *England*, I find that but 1 of 37 hath died *per An.* or 311 out of 11507, wherefore till I ſee another round Number, grounded upon many Obſervations, nearer than 30, I hope to have done pretty well in multiplying our Burials by 30, to find the Number of the People, the Product being 696360, and what we find by the Families they are 695,076, as aforeſaid.

The Third Way,

It was proved by *Grant*, that $\frac{1}{3}$ of the People died of the Plague, but *Anno* 1665 there died of the Plague near 98 Thouſand Perſons, the Quin-

Quintuple whereof is 490 Thouſand, as the Number of People in the Year 1665, whereunto adding above $\frac{1}{3}$, as the Increaſe between 1665 and 1686, the Total is 653 Thouſand, agreeing well enough with the other two Computations above-mentioned.

Wherefore let the Proportion of 1 to 30 continue till a better be put in its Place.

Memorandum, That 2 or 3 Hundred new Houſes would make a Contiguity of 2 or 3 other great Pariſhes, with the 134 already mentioned in the Bills of Mortality; and that an oval Wall of about 20 Miles in Compaſs would encloſe the ſame, and all the Shipping at *Deptford* and *Black-wall*, and would alſo fence in 20 Thouſand Acres of Land, and lay the Foundation or Deſignation of ſeveral vaſt Advantages to the Owners, and Inhabitants of that Ground, as alſo to the whole Nation and Government.

The FOURTH ESSAY.

Concerning the Proportions of People in the 8 eminent Cities of Chriſtendom undernamed, viz.

1. WE have by the Number of Burials in healthful Years, and by the Proportion of the living to thoſe who die yearly, as alſo by the Number of Houſes and Families within the 134 Pariſhes, called *London*, and the Eſtimate

Eſtimate of the Heads in each, pitch'd upon the Number of People in that City to be at a Medium 695718.

2. We have, by allowing that at *Paris* above 80 Thouſand Families (*viz.* 81280) do live in 23223 Houſes, 32 Palaces, and 38 Colleges, or that there are 81280 Kitchins within leſs than 24 Thouſand Street-doors; as alſo by allowing 30 Heads for every one that died neceſſarily there; we have pitch'd upon the Number of People there at a Medium to be 488055, nor have we reſtrained them to 300 Thouſand, by allowing with Monſieur *Auzout* 6 Heads for each of *Morery*'s 50000 Houſes or Families.

3. To *Amſterdam* we allow 187350 Souls, *viz.* 30 Times the Number of their Burials, which were 6245 in the Year 1685.

4. To *Venice* we allow 134 Thouſand Souls, as found there in a ſpecial Account taken by Authority, about 10 Years ſince, when the City abounded with ſuch as returned from *Candia*, then ſurrendered to the *Turks*.

5. To *Rome* we allow 119 Thouſand Chriſtians and 6000 Jews, in all 125 Thouſand Souls, according to an Account ſent hither of the ſame by Monſieur *Auzout*.

6. To *Dublin* we allow (as to *Amſterdam*) 30 Times its Burials, the Medium whereof for the laſt 2 Years is 2303, *viz.* 69090 Souls.

7. As to *Briſtol*, we ſay that if the 6400 Houſes of *Dublin* give 69,090 People, that the 5307 Houſes of *Briſtol*, muſt give above 56 Thouſand

Thouſand People; moreover, if the 29,325 Hearths of *Dublin*, give 69,090 People, the 16,752 Hearths of *Briſtol*, muſt give about 40 Thouſand; but the Medium of 56 Thouſand and 40 Thouſand is 48 Thouſand.

8. As for *Rouen*, we have no Help, but Monſieur *Auzout*'s Fancy of 80 Thouſand Souls to be in that City, and the Conjecture of knowing Men, that *Rouen* is between the $\frac{1}{7}$ and $\frac{1}{8}$ Part of *Paris*, and alſo that it is by a Third bigger than *Briſtol*: By all which, we eſtimate (till farther Light) that *Rouen* hath at moſt but 66 Thouſand People in it,

Now it may be wondered why we mentioned *Rouen* at all, having had ſo little Knowledge of it; whereunto we anſwer, that we did not think it juſt to compare *London* with *Paris*, as to Shipping and foreign Trade, without adding *Rouen* thereunto, *Rouen* being to *Paris* as that Part of *London* which is below the Bridge, is to what is above it.

All which we heartily ſubmit to the Correction of the Curious and Candid, in the mean Time obſerving according to the groſs Numbers under-mentioned.

	m.
London	696
Paris	488
Amſterdam	187
Venice	134
Rome	125
Dublin	69
Briſtol	48
Rouen	66

Ob-

Obſervations on the ſaid 8 *Cities.* *m.*

1. That the People of *Paris* being 488
of *Rome* 125
of *Rouen* 66

do make in all but 679
Thouſand, or 17 Thouſand leſs than the 696 Thouſand of *London* alone.

2. That the People of the 2 *Engliſh* Cities and Emporiums, *viz.* of *London* 696 Thouſand, and *Briſtol* 48 Thouſand, do make 744 Thouſand, or more than

	m.
In *Paris*	488
Amſterdam	187
Rouen	66
being in all	741 Thouſand

3. That the ſame 2 *Engliſh* Cities ſeem equivalent

To *Paris* which hath	488 Thouſand Souls.
To *Rouen*	66
To *Lyons*	100
To *Thoulouſe*	90
In all	744 Thouſand

If there be any Error in theſe Conjectures concerning theſe Cities of *France*, we hope they will be mended by thoſe whom we hear to be now at work upon that Matter.

4. That the King of *England*'s 3 Cities, *viz.*

Lon-

	m.			m.
London	696	exceed	*Paris*	488
Dublin	69		*Amſterdam*	187
Briſtol	48		*Venice*	134
In all	813		being but	809

5. That of the 4 great Emporiums, *London*, *Amſterdam*, *Venice* and *Rouen*, *London* alone is near double to the other 3, *viz.* above 7 to 4.

	m.	m.		m.
Amſterdam	187	387	*London*	696
Venice	134			
Rouen	66	2		
	774			

6. That *London* (for aught appears) is the greateſt and moſt conſiderable City of the World, but manifeſtly the greateſt Emporium.

When theſe Aſſertions have paſt the *Examen* of the Critics, we ſhall make another Eſſay, ſhewing how to apply thoſe Truths to the Honour and Profit of the King and Kingdom of *England*.

The FIFTH ESSAY.

Concerning Holland, *and the reſt of the United Provinces.*

SINCE the Cloſe of this Paper, it hath been objected from *Holland*, that what hath been ſaid of the Number of Houſes and People in *London*

London is not like to be true; for that if it were, then *London* would be $\frac{2}{3}$ of the whole Province of *Holland*. To which is anſwered, that *London* is the $\frac{2}{3}$ of all *Holland* and more, that Province having not a Million and 44000 Inhabitants (whereof 696 *m.* is the $\frac{2}{3}$) nor above 800 Thouſand, as we have credibly and often heard; for ſuppoſe *Amſterdam* hath, as we have elſewhere noted, 187000, the ſeven next great Cities at 30000 each one with another 210000, the 10 next at 15000 each 150000, the 10 ſmalleſt at 6000 each 60000, in all the 28 walled Cities and Towns of *Holland* 607000, in the Dorps and Villages 193000, which is about one Head for every 4 Acres of Land; whereas in *England* there is 8 Acres for every Head, without the Cities and Market Towns.

Now ſuppoſe *London* having 116000 Families, ſhould have 7 Heads in each, the Medium between Monſieur *Auzout* and *Grant*'s Reckonings, the Total of the People would be 812000, or if we reckon that there dies 1 out of 34 (the Medium between 30 and 37 above-mentioned) the Total of the People would be 34 Times 23212, *viz.* 789208, the Medium between which Number, and the above 812 Thouſand, is 800604, ſomewhat exceeding 800 Thouſand, the ſuppoſed Number of *Holland*.

Farthermore, I ſay that upon former Searches into the Peopling of the World, I never found that in any Country (not in *China* itſelf) there was more than one Man to every *Engliſh* Acre of

of Land (many Territories paſſing for well peo-pled, where there is but one Man for ten ſuch Acres) I found by meaſuring *Holland* and *Weſt-Frizia*, alias *North-Holland*, upon the beſt Maps, that it contained but as many ſuch Acres as *London* doth of People, *viz.* about 696000 Acres; I therefore venture to pronounce (till better informed) that the People of *London* are as many as thoſe of *Holland*, or at leaſt above $\frac{2}{3}$ of the ſame; which is enough to diſable the Objection above-mentioned; nor is there any Need to ſtrain up *London* from 696000 to 800000, tho' competent Reaſons have been given to that Purpoſe, and though the Author of the excellent Map of *London*, ſet forth *Anno* 1682, reckoned the People thereof (as by the ſaid Map appears) to be 1200,000, even when he thought the Houſes of the ſame to be but 85000.

The worthy Perſon who makes this Objection in the ſame Letter alſo ſaith,

1. That the Province of *Holland*, hath as many People as the other 6 United Provinces together, and as the whole Kingdom of *England*, and double to the City of *Paris* and its Suburbs; that is to ſay, 2 Millions of Souls. 2. He ſays that in *London* and *Amſterdam*, and other trading Cities there are 10 Heads to every Family, and that in *Amſterdam* there are not 22000 Families. 3. He excepteth againſt the Regiſter alledged by Monſieur *Auzout*, which makes 23223 Houſes, and above 80,000 Families to be in *Paris*, as alſo againſt the Regiſter alledged by *Petty*, making 105315 Houſes to be in *London*,

don, with a tenth Part of the ſame to be of Families more than Houſes, and probably will except againſt the Regiſter of 1163000 Houſes to be in all *England*, that Number giving at 6 ½ Heads to each Family, about 7 Millions of People; upon all which we remark as followeth, *viz.*

1. That if *Paris* doth contain but 488 thouſand Souls, that then all *Holland* containeth but the Double of that Number, or 976 Thouſand; wherefore *London* containing 696 Thouſand Souls, hath above ⅔ of all *Holland* by 46000.

2. If *Paris* containeth half as many People as there are in all *England*, it muſt contain 3 Millions and a half of Souls, or above 7 Times 488 Thouſand, and becauſe there do not die 20 Thouſand *per Ann.* out of *Paris*, there muſt die but one out of 175, whereas Monſieur *Auzout* thinks that there dies 1 out of 25, and there muſt live 149 Heads in every Houſe of *Paris* mentioned in the Regiſter, but there muſt be ſcarce 2 Heads in every Houſe of *England*; all which we think fit to be reconſidered.

I muſt as an *Engliſhman* take Notice of one Point more, which is, that theſe Aſſertions do reflect upon the Empire of *England*, for that it is ſaid, that *England* hath but two Millions of Inhabitants, and it might as well have been added, that *Scotland* and *Ireland*, with the Iſlands of *Man*, *Jerſey* and *Guernſey* have but ⅖ of the ſame Number, or 800 Thouſand more, or that all the King of *England*'s Subjects in *Europe* are but two Millions and 800 Thouſand Souls; whereas he ſaith, that the Subjects of the 7 United

nited Provinces are four Millions. To which we anſwer, that the Subjects of the ſaid 7 Provinces, are by this Objector's own ſhewing, but the Quadruple of *Paris*, or 1932 Thouſand Souls, *Paris* containing but 488000 as afore hath been prov'd, and we do here affirm that *England* hath 7 Millions of People, and that *Scotland*, *Ireland*, with the Iſlands of *Man*, *Jerſey* and *Guernſey*, hath $\frac{2}{5}$ of the ſaid Number, or two Millions 800 Thouſand more, in all 9 Millions 800 Thouſand; whereas by the Objector's Doctrine, if the 7 Provinces have 1932 Thouſand People, the King of *England*'s Territories ſhould have but $\frac{7}{10}$ of the ſame Number, *viz.* 1351 Thouſand; whereas we ſay 9800 Thouſand, as aforeſaid, which Difference is ſo groſs as that it deſerves to be thus reflected upon.

To conclude, we expect from the concerned Critics of the World, that they would prove,

1. That *Holland* and *Weſt-Frizia*, and the 28 Towns and Cities thereof, hath more People than *London* alone.

2. That any 3 the beſt Cities of *France*, any 2 of all Chriſtendom, or any 1 of the World, hath the ſame, or better Houſing, and more foreign Trade than *London*, even in the Year that King *James* II. came to the Empire thereof.

POLI-

POLITICAL ARITHMETICK,

OR A

DISCOURSE

Concerning

The Extent and Value of Lands, People, Buildings; Husbandry, Manufactures, Commerce, Fishery, Artizans, Seamen, Soldiers; Publick Revenues, Interest, Taxes, Superlucration, Registries, Banks; Valuation of Men, Increasing of Seamen, of Militia's, Harbours, Situation, Shipping, Power at Sea, &c. As the same relates to every Country in general, but more particularly to the Territories of his Majesty of *Great Britain*, and his Neighbours of *Holland*, *Zealand*, and *France*.

By Sir WILLIAM PETTY, *late Fellow of the* Royal Society.

1691.

LET this Book called *Political Arithmetick*, which was long ſince wrote by Sir *William Petty* deceaſed, be Printed.

Given at the Court at Whitehall *the 7th Day of* November, 1690.

Nottingham.

TO THE

KING'S

Moſt Excellent MAJESTY.

SIR,

WHILST every one meditates ſome fit Offering for your Majeſty, ſuch as may beſt agree with your happy Exaltation to this Throne; I preſume to offer, what my Father long ſince wrote, to ſhew the Weight and Importance of the *Engliſh* Crown.

It was by him ſtiled *Political Arithmetick*, inaſmuch as Things of Government, and of no leſs Concern and Extent, than the Glory of the Prince, and the Happineſs and Greatneſs of the People, are, by the ordinary Rules of Arithmetick, brought into a Sort of Demonſtration. He was allowed by all, to be the Inventor of this Method of Inſtruction; where the perplexed and intricate Ways of the World, are explained by a very mean Piece of Science; and had not the Doctrines of this Eſſay

offended *France*, they had long ſince ſeen the Light, and had found Followers, as well as Improvements before this Time, to the Advantage perhaps of Mankind.

But this has been reſerved to the Felicity of your Majeſty's Reign, and to the Expectation which the Learned have therein; and if while in this, I do ſome Honour to the Memory of a good Father, I can alſo pay Service, and ſome Teſtimony of my Zeal and Reverence to ſo great a King, it will be the utmoſt Ambition of

SIR,

Your Majeſty's moſt dutiful

and moſt obedient Subject,

SHELBORNE.

PRE-

PREFACE.

FORASMUCH as Men, who are in a decaying Condition, or who have but an ill Opinion of their own Concernments, inſtead of being (as ſome think) the more induſtrious to reſiſt the Evils they apprehend, do contrariwiſe become the more languid and ineffectual in all their Endeavours, neither caring to attempt or proſecute even the probable Means of their Relief. Upon this Conſideration, as a Member of the Commonwealth, next to knowing the preciſe Truth in what Condition the common Intereſt ſtands, I would in all doubtful Caſes think the beſt, and conſequently not deſpair, without ſtrong and manifeſt Reaſons, carefully examining whatever tends to leſſen my Hopes of the publick Welfare.

I have therefore thought fit to examine the following Perſuaſions, which I find too current in the World, and too much to have affected the Minds of ſome, to the Prejudice of all, *viz.*

That the Rents of Lands are generally fallen; that therefore, and for many other Reaſons, the

whole Kingdom grows every Day poorer and poorer; that formerly it abounded with Gold, but now there is a great Scarcity both of Gold and Silver; that there is no Trade nor Employment for the People, and yet that the Land is under-peopled; that Taxes have been many and great; that *Ireland* and the Plantations in *America*, and other Additions to the Crown, are a Burthen to *England*; that *Scotland* is of no Advantage; that Trade in general doth lamentably decay; that the *Hollanders* are at our Heels, in the Race of Naval Power; the *French* grow too faſt upon both, and appear ſo rich and potent, that it is but their Clemency that they do not devour their Neighbours; and finally, that the Church and State of *England*, are in the ſame Danger with the Trade of *England*; with many other diſmal Suggeſtions, which I had rather ſtifle than repeat.

'Tis true, the Expence of foreign Commodities hath of late been too great; much of our Plate, had it remain'd Money, would have better ſerved Trade; too many Matters have been regulated by Laws, which Nature, long Cuſtom, and general Conſent, ought only to have governed; the Slaughter and Deſtruction of Men by the late Civil Wars and Plague have been great; the Fire at *London*, and Diſaſter at *Chatham*, have begotten Opinions in the *Vulgus* of the World to our Prejudice; the Nonconformiſts increaſe; the People of *Ireland* think

long

long of their Settlement; the *Engliſh* there apprehend themſelves to be Aliens, and are forced to ſeek a Trade with Foreigners, which they might as well maintain with their own Relations in *England*. But notwithſtanding all this, (the like whereof was always in all Places), the Buildings of *London* grow great and glorious; the *American* Plantations employ four Hundred Sail of Ships; Actions in the *Eaſt-India* Company are near double the principal Money; thoſe who can give good Security, may have Money under the Statute-Intereſt; Materials for building, (even Oaken-Timber) are little the dearer, ſome cheaper for the rebuilding of *London*; the Exchange ſeems as full of Merchants as formerly; no more Beggars in the Streets, nor executed for Thieves, than heretofore; the Number of Coaches, and Splendor of Equipage exceeding former Times; the publick Theatres very magnificent: the King has a greater Navy, and ſtronger Guards than before our Calamities; the Clergy rich, and the Cathedrals in Repair; much Land has been improved, and the Price of Food ſo reaſonable, as that Men refuſe to have it cheaper, by admitting of *Iriſh* Cattle; and in brief, no Man needs to want that will take moderate Pains. That ſome are poorer than others, ever was and ever will be: And that many are naturally querulous and envious, is an Evil as old as the World.

Theſe general Obſervations, and that Men eat, and drink, and laugh as they uſe to do,

have encouraged me to try if I could alſo comfort others, being ſatisfied myſelf, that the Intereſt and Affairs of *England* are in no deplorable Condition.

The Method I take to do this, is not yet very uſual; for inſtead of uſing only comparative and ſuperlative Words, and intellectual Arguments, I have taken the Courſe (as a Specimen of the Political Arithmetick I have long aimed at) to expreſs myſelf in Terms of *Number*, *Weight*, or *Meaſure*; to uſe only Arguments of Senſe, and to conſider only ſuch Cauſes, as have viſible Foundations in Nature; leaving thoſe that depend upon the mutable Minds, Opinions, Appetites, and Paſſions of particular Men, to the Conſideration of others: Really profeſſing myſelf as unable to ſpeak ſatisfactorily upon thoſe Grounds (if they may be called Grounds,) as to foretell the Caſt of a Dye; to play well at Tennis, Billiards, or Bowls, (without long Practice,) by Virtue of the moſt elaborate Conceptions that ever have been written *De Projectilibus & Miſſilibus*, or of the Angles of Incidence and Reflection.

Now the Obſervations or Poſitions expreſſed by *Number*, *Weight*, and *Meaſure*, upon which I bottom the enſuing Diſcourſes, are either true, or not apparently falſe, and which if they are not already true, certain, and evident, yet may be made ſo by the Sovereign Power, *Nam id certum eſt quod certum reddi poteſt*, and if they are falſe, not ſo falſe as to deſtroy the Argument

ment they are brought for; but at worſt are ſufficient as Suppoſitions to ſhew the Way to that Knowledge I aim at. And I have withal for the preſent confined myſelf to the ten principal Concluſions hereafter particularly handled, which if they ſhall be judged material, and worthy of a better Diſcuſſion, I hope all ingenious and candid Perſons will rectify the Errors, Defects, and Imperfections, which probably may be found in any of the Poſitions, upon which theſe Ratiocinations were grounded. Nor would it miſbecome Authority itſelf, to clear the Truth of thoſe Matters which private Endeavours cannot reach to,

CHAP.

CHAP. I.

That a ſmall Country and few People, by its Situation, Trade, and Policy, may be equivalent in Wealth and Strength, to a far greater People and Territory: And particularly that Conveniencies for Shipping and Water-Carriage, do moſt eminently and fundamentally conduce thereunto.

THE firſt principal Concluſion by Reaſon of its Length, I conſider in three Parts; whereof the firſt is, That a ſmall Country and few People, may be equivalent in Wealth and Strength to a far greater People and Territory.

This Part of the firſt principal Concluſion needs little Proof; foraſmuch as one Acre of Land, may bear as much Corn and feed as many Cattle as Twenty, by the Difference of the Soil; ſome Parcel of Ground is naturally ſo defenſible, as that an hundred Men being poſſeſſed thereof, can reſiſt the Invaſion of five Hundred; and bad Land may be improved and made good; Bog may by draining be made Meadow; Heath-land may (as in *Flanders*) be made to bear Flax and Clover-graſs, ſo as to advance in Value from One to an Hundred; the ſame Land being built upon, may centuple the Rent which it yielded as Paſture; one Man is more nimble or ſtrong, and more patient of Labour than another; one Man by Art may do

as

as much Work, as many without it; *viz.* one Man with a Mill can grind as much Corn, as Twenty can pound in a Mortar; one Printer can make as many Copies, as an hundred Men can write by Hand; one Horſe can carry upon Wheels, as much as Five upon their Backs; and in a Boat, or upon Ice, as Twenty: So that I ſay again, this firſt Point of this general Poſition, needs little or no Proof. But the ſecond and more material Part of this Concluſion is, that this Difference in Land and People, ariſes principally from their Situation, Trade, and Policy.

To clear this, I ſhall compare *Holland* and *Zealand*, with the Kingdom of *France*, *viz.* *Holland* and *Zealand* do not contain above one Million of *Engliſh* Acres, whereas the Kingdom of *France* contains above 80.

Now the original and primitive Difference holds Proportion as Land to Land, for it is hard to ſay, that when theſe Places were firſt planted, whether an Acre in *France* was better than the like Quantity in *Holland* and *Zealand*; nor is there any Reaſon to ſuppoſe, but that therefore upon the firſt Plantation, the Number of Planters was in Proportion to the Quantity of Land; wherefore, if the People are not in the ſame Proportion as the Land, the ſame muſt be attributed to the Situation of the Land, and to the Trade and Policy of the People ſuperſtructed thereupon.

The next Thing to be ſhewn is, that *Holland* and *Zealand* at this Day, is not only an eightieth Part

Part as rich and ſtrong as *France*, but that it hath advanced to one Third or thereabouts, which I think will appear upon the Balance of the following Particulars, *viz.*

As to the Wealth of *France*, a certain Map of that Kingdom, ſet forth *Anno* 1647, repreſents it to be fifteen Millions, whereof ſix did belong to the Church, the Author thereof (as I ſuppoſe) meaning the Rents of the Lands only: And the Author of a moſt judicious Diſcourſe of Huſbandry (ſuppoſed to be Sir *Richard Weſton*,) doth from Reaſon and Experience ſhew, that Lands in the Netherlands, by bearing Flax, Turneps, Clover-graſs, Madder, *&c.* will eaſily yield 10 *l. per* Acre; ſo as the Territories of *Holland* and *Zealand*, ſhould by his Account yield at leaſt Ten Millions *per Annum*, yet I do not believe the ſame to be ſo much, nor *France* ſo little as aboveſaid, but rather, that one bears to the other as about 7, or 8 to 1.

The People of *Amſterdam*, are one Third of thoſe in *Paris* or *London*, which two Cities differ not in People a twentieth Part from each other, as hath appeared by the Bills of Burials and Chriſtenings for each. But the Value of the Buildings in *Amſterdam*, may well be half that of *Paris*, by Reaſon of the Foundations, Grafts, and Bridges, which in *Amſterdam* are more numerous and chargeable than at *Paris*. Moreover the Habitations of the pooreſt People in *Holland* and *Zealand*, are twice or thrice as good as thoſe of *France*; but the People of the one

one to the People of the other, being but as 13 to 1, the Value of the Houſing muſt be as about 5 to 1.

The Value of the Shipping of *Europe*, being about two Millions of Tons, I ſuppoſe the *Engliſh* have Five Hundred Thouſand, the *Dutch* Nine Hundred Thouſand, the *French* an Hundred Thouſand, the *Hamburgers*, and the Subjects of *Denmark*, *Sweden*, and the Town of *Dantzick* Two Hundred and Fifty Thouſand, and *Spain*, *Portugal*, *Italy*, &c. Two Hundred and Fifty Thouſand; ſo as the Shipping in our Caſe of *France* to that of *Holland*, is about 1 to 9, which reckoned as great and ſmall, new and old, one with another at 8 *l. per* Ton, makes the Worth to be as Eight Hundred Thouſand Pounds, to Seven Millions and two Hundred Thouſand Pounds. The *Hollanders* Capital in the *Eaſt-India* Company, is worth above Three Millions, where the *French* as yet have little or nothing.

The Value of the Goods exported out of *France* into all Parts, are ſuppoſed Quadruple to what is ſent to *England* alone, and conſequently in all about Five Millions, but what is exported out of *Holland* into *England* is worth 2 Millions; and what is exported thence into all the World beſides, is Sextuple to the ſame.

The Monies yearly raiſed by the King of *France*, as the ſame appears by the Book intituled *The State of* France, dedicated to the King, printed *Anno* 1669, and ſet forth ſeveral Times by Authority, is 82000000 of *French* Livres, which

which is about 6½ Millions of Pounds Sterling, of which Sum the Author ſays, that one fifth Part was abated for Non-valeurs or Inſolvencies, ſo (as I ſuppoſe) not above Five Millions were effectually raiſed: But whereas ſome ſay, that the King of *France* raiſed Eleven Millions as the ½ of the Effects of *France*; I humbly affirm, that all the Land and Sea Forces, all the Buildings and Entertainments, which we have heard by common Fame, to have been ſet forth and made in any of theſe ſeven laſt Years, needed not to have coſt ſix Millions Sterling; wherefore, I ſuppoſe he hath not raiſed more, eſpecially ſince there were one fifth Inſolvencies, when the Tax was at that Pitch. But *Holland* and *Zealand*, paying 67 of the 100, paid by all the United Provinces, and the City of *Amſterdam* paying 27 of the ſaid 67; it follows that if *Amſterdam* hath paid 4000 *l.* Flemiſh *per Diem*, or about 1400000 *l. per Annum*, or 800000 *l.* Sterling; that all *Holland* and *Zealand* have paid 2100000 *l. per Annum*: Now the Reaſons why I think they pay ſo much, are theſe, *viz.*

1. The Author of the State of the *Netherlands* ſaith ſo.

2. Exciſe of Victual at *Amſterdam*, ſeems above half the original Value of the ſame, *viz.*

Ground Corn pays 20 Stivers the Buſhel, or 63 Gilders the Laſt; Beer 113 Stivers the Barrel, Houſing ½ of the Rent, Fruit ⅛ of what it coſt; other Commodities ⅐, ⅛, 1/9, 1/12; Salt *ad libitum*, all weighed Goods pay beſides the Premiſſes

miſſes a vaſt Sum; now if the Expence of the People of *Amſterdam* at a Medium, and without Exciſe were 8 *l. per Annum*, whereas in *England* 'tis 7 *l.* then if all the ſeveral Impoſts above named, raiſe it Five Pound more, there being 160000 Souls in *Amſterdam*, the Sum of 800000 Pound Sterling *per Annum* will thereby be raiſed.

3. Though the Expence of each Head ſhould be 13 *l. per Annum*; 'tis well known that there be few in *Amſterdam*, who do not earn much more than the ſaid Expence.

4. If *Holland* and *Zealand* pay *per Annum* 2100000 *l.* then all the Provinces together, muſt pay about 3000000 *l.* leſs than which Sum *per An.* perhaps is not ſufficient to have maintained the Naval War with *England*, 72000 Land Forces, beſides all other the ordinary Charges of their Government, whereof the Church is there a Part: To conclude, it ſeems from the Premiſſes, that all *France* doth not raiſe above thrice as much from the publick Charge, as *Holland* and *Zealand* alone do.

5. Intereſt of Money in *France*, is 7 *l. per Cent.* but in *Holland* ſcarce half ſo much.

6. The Countries of *Holland* and *Zealand* conſiſting as it were of Iſlands guarded with the Sea, Shipping, and Marſhes, is defenſible at one Fourth of the Charge, that a plain open Country is, and where the Seat of War may be both Winter and Summer; whereas in the others, little can be done but in the Summer only.

7. But

7. But above all the Particulars hitherto considered, that of Superlucration ought chiefly to be taken in; for if a Prince have never so many Subjects, and his Country be never so good, yet if either through Sloth, or extravagant Expences, or Oppression and Injustice, whatever is gained shall be spent as fast as gotten, that State must be accounted poor; wherefore let it be considered, how much, or how many Times rather, *Holland* and *Zealand* are now above what they were 100 Years ago, which we must also do of *France:* Now if *France* hath scarce doubled its Wealth and Power, and that the other have decupled theirs, I shall give the Preference to the latter, even although the $\frac{9}{10}$ increased by the one, should not exceed the one half gained by the other, because one has a Store for nine Years, the other but for one.

To conclude, upon the Whole it seems, that though *France* be in People to *Holland* and *Zealand* as 13 to 1, and in Quantity of good Land, as 80 to 1, yet is not 13 Times richer and stronger, much less 80 Times, nor much above thrice, which was to be proved.

Having thus dispatched the two first Branches of the first principal Conclusion, it follows, to shew that this Difference of Improvement in Wealth and Strength, arises from the Situation, Trade, and Policy of the Places respectively, and in particular from Conveniencies for Shipping and Water-Carriage.

Many

Many writing on this Subject do ſo magnify the *Hollanders* as if they were more, and all other Nations leſs than Men (as to the Matters of Trade and Policy) making them Angels, and others Fools, Brutes, and Sots, as to thoſe Particulars; whereas I take the Foundation of their Atchievements to lie originally in the Situation of the Country, whereby they do Things inimitable by others, and have Advantages whereof others are incapable.

Firſt, The Soil of *Holland* and *Zealand* is low Land, rich and fertile; whereby it is able to feed many Men, and ſo as that Men may live near each other, for their mutual Aſſiſtance in Trade. I ſay that a Thouſand Acres, that can feed 1000 Souls, is better than 10000 Acres of no more Effect, for the following Reaſons, *viz.*

1. Suppoſe ſome great Fabrick were in building by a Thouſand Men, ſhall not much more Time be ſpared if they lived all upon a Thouſand Acres, than if they were forced to live upon ten Times as large a Scope of Land.

2. The Charge of the Cure of their Souls, and the Miniſtry would be far greater in one Caſe than in the other; as alſo of mutual Defence in Caſe of Invaſion, and even of Thieves and Robbers: Moreover the Charge of the Adminiſtration of Juſtice would be much eaſier, where Witneſſes and Parties may be eaſily ſummoned, Attendance leſs expenſive, when Mens Actions would be better known, when Wrongs and Injuries could not be covered, as in thin peopled Places they are.

Laſtly, thoſe who live in ſolitary Places, muſt be their own Soldiers, Divines, Phyſicians, and Lawyers, and muſt have their Houſes ſtored with neceſſary Proviſions, (like a Ship going upon a long Voyage,) to the great Waſte, and needleſs Expence of ſuch Proviſions; the Value of this firſt Convenience to the *Dutch*, I reckon or eſtimate to be about 100000 *l. per An.*

2dly, *Holland* is a level Country, ſo as in any Part thereof a Windmill may be ſet up, and by its being moiſt and vaporous, there is always Wind ſtirring over it, by which Advantage the Labour of many Thouſand Hands is ſaved, foraſmuch as a Mill made by one Man in half a Year, will do as much Labour, as four Men for five Years together. This Advantage is greater or leſs, where Employment or Eaſe of Labour is ſo; but in *Holland* 'tis eminently great, and the Worth of this Conveniency is near 150000 *l.*

3dly, There is much more to be gained by Manufacture than Huſbandry, and by Merchandize than Manufacture; but *Holland* and *Zealand*, being ſeated at the Mouths of three long great Rivers, and paſſing through rich Countries, do keep all the Inhabitants upon the Sides of thoſe Rivers but as Huſbandmen, whilſt themſelves are the Manufactors of their Commodities, and do diſpenſe them into all Parts of the World, making Returns for the ſame, at what Prices almoſt they pleaſe themſelves; and in ſhort, they keep the Keys of Trade of thoſe Countries, through which the ſaid Rivers paſs; the

the Value of this third Conveniency, I ſuppoſe to be 200000 *l.*

4thly, In *Holland* and *Zealand*, there is ſcarce any Place of Work, or Buſineſs, one Mile diſtant from a navigable Water, and the Charge of Water-Carriage is generally but $\frac{1}{15}$ or $\frac{1}{20}$ Part of Land-Carriage; wherefore if there be as much Trade there as in *France*, then the *Hollanders* can out-ſell the *French* $\frac{14}{15}$ of all the Expence, of all Travelling, Poſtage, and Carriage, whatſoever, which even in *England* I take to be 300000 Pounds *per Annum*, where the very Poſtage of Letters coſts the People perhaps 50000 *l. per Annum*, though farmed at much leſs, and all other Labour of Horſes, and Porters, at leaſt ſix Times as much: The Value of this Conveniency I eſtimate to be above 300000 *l. per An.*

5. The Defenſibleneſs of the Country, by Reaſon of its Situation in the Sea upon Iſlands, and in the Marſhes, impaſſible Ground dyked and trenched, eſpecially conſidering how that Place is aimed at for its Wealth; I ſay the Charge of defending that Country is eaſier than if it were a plain Champian, at leaſt 200000 *l. per Annum.*

6. *Holland* is ſo conſiderable for keeping Ships in Harbour with ſmall Expence of Men, and Ground Tackle, that it ſaves *per An.* 200000 *l.* of what muſt be ſpent in *France*. Now if all theſe natural Advantages do amount to above 1 Million *per Annum* Profits, and that the Trade of all *Europe*, nay of the whole World, with

which our *Europeans* do trade, is not above 45 Millions *per Annum*, and if $\frac{1}{50}$ of the Value be $\frac{1}{7}$ of the Profit, it is plain that the *Hollander* may command and govern the whole Trade.

7. Thoſe who have their Situation thus towards the Sea, and abound with Fiſh at home, and having alſo the Command of Shipping, have by Conſequence the Fiſhing Trade, whereof that of Herrings alone brings more yearly Profit to the *Hollanders* than the Trade of the *Weſt-Indies* to *Spain*, or of the *Eaſt* to themſelves, as many have affirmed, being as the ſame ſay, *viis & modis*, of above 3 Millions *per An.* Profit.

8. It is not to be doubted, but thoſe who have the Trade of Shipping and Fiſhing, will ſecure themſelves of the Trade of Timber for Ships, Boats, Maſts, and Caſks; of Hemp for Cordage, Sails, and Nets; of Salt; of Iron; as alſo of Pitch, Tar, Roſin, Brimſtone, Oil, and Tallow, as neceſſary Appurtenances to Shipping and Fiſhing.

9. Thoſe who predominate in Shipping and Fiſhing, have more Occaſions than others to frequent all Parts of the World, and to obſerve what is wanting or redundant every where, and what each People can do, and what they deſire, and conſequently to be the Factors and Carriers for the whole World of Trade. Upon which Ground they bring all native Commodities to be manufactured at home, and carry the ſame back, even to that Country in which they grew; all which we ſee.

For,

For, do they not work the Sugars of the *Weſt-Indies?* The Timber and Iron of the *Baltick?* The Hemp of *Ruſſia?* The Lead, Tin, and Wool of *England?* The Quickſilver and Silk of *Italy?* The Yarns, and Dying Stuffs of *Turkey*, &c. To be ſhort, in all the ancient States, and Empires, thoſe who had the Shipping, had the Wealth, and if 2 *per Cent.* in the Price of Commodities, be perhaps 20 *per Cent.* in the Gain; it is manifeſt that they who can in 45 Millions, under-ſell others by one Million, (upon Account of natural and intrinſick Advantages only) may eaſily have the Trade of the World without ſuch angelical Wits and Judgments, as ſome attribute to the *Hollanders.*

Having thus done with their Situation, I come now to their Trade.

It is commonly ſeen, that each Country flouriſheth in the Manufacture of its own native Commodities, *viz. England* for woollen Manufacture, *France* for Paper, *Luic-land* for Iron-Ware, *Portugal* for Confectures, *Italy* for Silks; upon which Principle it follows, that *Holland* and *Zealand* muſt flouriſh moſt in the Trade of Shipping, and ſo become Carriers and Factors of the whole World of Trade. Now the Advantages of the Shipping Trade are as followeth, *viz.*

Huſbandmen, Seamen, Soldiers, Artizans, and Merchants, are the very Pillars of any Commonwealth; all the other great Profeſſions do riſe out of the Infirmities and Miſcarriages of

theſe; now the Seaman is three of theſe four. For every Seaman of Induſtry and Ingenuity, is not only a Navigator, but a Merchant, and alſo a Soldier; not becauſe he hath often Occaſion to fight, and handle Arms; but becauſe he is familiarized with Hardſhip and Hazards, extending to Life and Limbs; for Training and Drilling is a ſmall Part of Soldiery, in reſpect of this laſt-mentioned Qualification; the one being quickly and preſently learned, the other not without many Years moſt painful Experience: Wherefore to have the Occaſion of abounding in Seamen, is a vaſt Conveniency.

2. The Huſbandman of *England* earns but about 4*s.* *per* Week, but the Seamen have as good as 12 *s.* in Wages, Victuals (and as it were Houſing) with other Accommodations, ſo as a Seaman is in Effect three Huſbandmen; wherefore there is little Ploughing, and Sowing of Corn in *Holland* and *Zealand*, or breeding of young Cattle: But their Land is improved by building Houſes, Ships, Engines, Dykes, Wharfs, Gardens of Pleaſure, extraordinary Flowers and Fruits; for Dairy and feeding of Cattle, for Rape, Flax, Madder, *&c.* The Foundations of ſeveral advantageous Manufactures.

3. Whereas the Employment of other Men is confined to their own Country, that of Seamen is free to the whole World; ſo as where Trade may (as they call it) be dead here or there, now and then, it is certain that ſome where or other in the World, Trade is always quick enough,

nough, and Proviſions are always plentiful, the Benefit whereof thoſe who command the Shipping enjoy, and they only.

4. The great and ultimate Effect of Trade is not Wealth at large, but particularly Abundance of Silver, Gold, and Jewels, which are not periſhable, nor ſo mutable as other Commodities, but are Wealth at all Times, and all Places.: Whereas Abundance of Wine, Corn, Fowls, Fleſh, *&c.* are Riches but *hic & nunc*, ſo as the raiſing of ſuch Commodities, and the following of ſuch Trade, which does ſtore the Country with Gold, Silver, Jewels, *&c.* is profitable before others. But the Labour of Seamen, and Freight of Ships, is always of the Nature of an exported Commodity, the Overplus whereof, above what is imported, brings home Money, *&c.*

5. Thoſe who have the Command of the Sea Trade, may work at eaſier Freight with more Profit, than others at greater, for as Cloth muſt be cheaper made, when one cards, another ſpins, another weaves, another draws, another dreſſes, another preſſes and packs; than when all the Operations abovementioned, were clumſily performed by the ſame Hand; ſo thoſe who command the Trade of Shipping, can build long ſlight Ships for carrying Maſts, Fir-Timber, Boards, Balks, *&c.* and ſhort ones for Lead, Iron, Stones, *&c.* One Sort of Veſſels to trade at Ports where they need never lie aground, others where they muſt jump upon the

Sand twice every 12 Hours; one Sort of Veſſels, and Way of manning in Time of Peace, and for cheap groſs Goods, another for War and precious Commodities; one Sort of Veſſels for the turbulent Sea, another for Inland Waters and Rivers; one Sort of Veſſels and Rigging, where Haſte is requiſite for the Maidenhead of a Market, another where $\frac{1}{5}$ or $\frac{1}{4}$ Part of the Time makes no Matter. One Sort of Maſting and Rigging for long Voyages, another for Coaſting. One Sort of Veſſels for Fiſhing, another for Trade. One Sort for War for this or that Country, another for Burthen only. Some for Oars, ſome for Poles, ſome for Sails, and ſome for Draught by Men or Horſes, ſome for the Northern Navigations amongſt Ice, and ſome for the South againſt Worms, *&c.* And this I take to be the chief of ſeveral Reaſons, why the *Hollanders* can go at leſs Freight than their Neighbours, *viz.* becauſe they can afford a particular Sort of Veſſels for each particular Trade.

I have ſhewn how Situation hath given them Shipping, and how Shipping hath given them in Effect all other Trade, and how foreign Traffick muſt give them as much Manufacture as they can manage themſelves, and as for the Overplus make the reſt of the World but as Workmen to their Shops. It now remains to ſhew the Effects of their Policy, ſuperſtructed upon theſe natural Advantages, and not as ſome think upon the Exceſs of their Underſtandings.

I have

I have omitted to mention that the *Hollanders* were one hundred Years ſince, a poor and oppreſſed People, living in a Country naturally cold, moiſt, and unpleaſant; and were withal perſecuted for their Heterodoxy in Religion.

From hence it neceſſarily follows, that this People muſt labour hard, and ſet all Hands to work: Rich and Poor, Young and Old, muſt ſtudy the Art of Number, Weight, and Meaſure; muſt fare hard, provide for Impotents, and for Orphans, out of Hope to make Profit by their Labours; muſt puniſh the Lazy by Labour, and not by cripling them: I ſay, all theſe Particulars, ſaid to be the ſubtile Excogitations of the *Hollanders*, ſeem to me but what could not almoſt have been otherwiſe.

Liberty of Conſcience, Regiſtry of Conveyances, ſmall Cuſtoms, Banks, Lumbards, and Law Merchant, riſe all from the ſame Spring, and tend to the ſame Sea; as for Lowneſs of Intereſt, it is alſo a neceſſary Effect of all the Premiſſes, and not the Fruit of their Contrivance.

Wherefore we ſhall only ſhew in particular the Efficacy of each, and firſt of Liberty of Conſcience; but before I enter upon theſe, I ſhall mention a Practice almoſt forgotten (whether it referreth to Trade or Policy is not material,) which is, the *Hollanders* under-maſting and ſailing ſuch of their Shipping, as carry cheap and groſs Goods, and whoſe Sale doth not depend much upon Seaſon.

It

It is to be noted, that of two equal and like Veſſels, if one ſpreads 1600 Yards of like Canvaſe, and the other 2500, their Speed is but as four to five, ſo as one brings home the ſame Timber in four Days, as the other will in five. Now if we conſider that although thoſe Ships be but four or five Days under Sail, that they are perhaps thirty upon the Voyage; ſo as the one is but $\frac{1}{30}$ Part longer upon the whole Voyage than the other, though $\frac{1}{5}$ longer under Sail. Now if Maſts, Yards, Rigging, Cables, and Anchors, do all depend upon the Quantity and Extent of the Sails, and conſequently Hands alſo; it follows that the one Veſſel goes at one third leſs Charge, loſing but one thirtieth of the Time, and of what depends thereupon.

I now come to the firſt Policy of the *Dutch*, *viz.* Liberty of Conſcience; which I conceive they grant upon theſe Grounds, (But keeping up always a Force to maintain the Common Peace,) 1. They themſelves broke with *Spain*, to avoid the Impoſition of the Clergy. 2. Diſſenters of this Kind, are for the moſt Part, thinking, ſober, and patient Men. and ſuch as believe that Labour and Induſtry is their Duty towards God. (How erroneous ſoever their Opinions be.) 3. Theſe People believing the Juſtice of God, and ſeeing the moſt licentious Perſons to enjoy moſt of the World, and its beſt Things, will never venture to be of the ſame Religion and Profeſſion with Voluptuaries, and Men of extreme Wealth and Power, who they think have their Portion in this World.

4. They

4. They cannot but know, that no Man can believe what himſelf pleaſes, and to force Men to ſay they believe what they do not, is vain, abſurd, and without Honour to God.

5. The *Hollanders* knowing themſelves not to be an infallible Church, and that others had the ſame Scripture for Guides as themſelves, and withal the ſame Intereſt to ſave their Souls, did not think fit to make this Matter their Buſineſs; not more than to take Bonds of the Seamen they employ, not to caſt away their own Ships and Lives.

6. The *Hollanders* obſerve that in *France* and *Spain*, (eſpecially the latter) the Churchmen are about one hundred to one, to what they uſe or need; the principal Care of whom is to preſerve Uniformity, and this they take to be a ſuperfluous Charge.

7. They obſerve where moſt Endeavours have been uſed to keep Uniformity, there Heterodoxy hath moſt abounded.

8. They believe that if $\frac{1}{4}$ of the People were heterodox, and that if that whole Quarter ſhould by Miracle be removed, that within a ſmall Time one Quarter of the Remainder would again become heterodox ſome Way or other, it being natural for Men to differ in Opinion in Matters above Senſe and Reaſon; and for thoſe who have leſs Wealth, to think they have the more Wit and Underſtanding, eſpecially of the Things of God, which they think chiefly belong to the Poor.

9. They

9. They think the Caſe of the primitive Chriſtians, as it is repreſented in the *Acts of the Apoſtles*, looks like that of the preſent Diſſenters, (I mean externally.) Moreover it is to be obſerved that Trade doth not (as ſome think) beſt flouriſh under popular Governments, but rather that Trade is moſt vigorouſly carried on, in every State and Government, by the heterodox Part of the ſame, and ſuch as profeſs Opinions different from what are publickly eſtabliſhed: (that is to ſay) in *India* where the *Mahometan* Religion is authorized, there the *Banians* are the moſt conſiderable Merchants. In the *Turkiſh* Empire the *Jews* and Chriſtians. At *Venice*, *Naples*, *Leghorn*, *Genoa*, and *Liſbon*, *Jews*, and Non-Papiſt Merchant-Strangers: But to be ſhort, in that Part of *Europe*, where the Roman Catholick Religion now hath, or lately hath had Eſtabliſhment, there three Quarters of the whole Trade is in the Hands of ſuch as have ſeparated from that Church (that is to ſay) the Inhabitants of *England*, *Scotland*, and *Ireland*, as alſo thoſe of the *United Provinces*, with *Denmark*, *Sweden*, and *Norway*, together with the Subjects of the *German* Proteſtant Princes, and the *Hans* Towns, do at this Day poſſeſs three Quarters of the Trade of the World; and even in *France* itſelf, the *Hugonots* are proportionably far the greateſt Traders; nor is it to be denied but that in *Ireland*, where the ſaid *Roman* Religion is not authorized, there the Profeſſors thereof have a great Part of the Trade.

From

From whence it follows that Trade is not fixt to any Species of Religion as ſuch; but rather as before hath been ſaid to the heterodox Part of the whole, the Truth whereof appears alſo in all the particular Towns of greateſt Trade in *England*; nor do I find Reaſon to believe, that the *Roman* Catholick Seamen in the whole World, are ſufficient to man effectually a Fleet equal to what the King of *England* now hath; but the Non-papiſt Seamen can do above thrice as much. Wherefore he whom this latter Party doth affectionately own to be their Head, cannot probably be wronged in his Sea-concernments by the other; from whence it follows, that for the Advancement of Trade, (if that be a ſufficient Reaſon) Indulgence muſt be granted in Matters of Opinion; though licentious Actings as even in *Holland*, be reſtrained by Force.

The ſecond Policy or Help to Trade uſed by the *Hollanders*, is ſecuring the Titles to Lands and Houſes; for although Lands and Houſes may be called *Terra firma & res immobilis*, yet the Title unto them is no more certain than it pleaſes the Lawyers and Authority to make them; wherefore the *Hollanders* do by Regiſtries, and other Ways of Aſſurance, make the Title as immoveable as the Lands, for there can be no Encouragement to Induſtry, where there is no Aſſurance of what ſhall be gotten by it; and where by Fraud and Corruption, one Man may take away with Eaſe and by a Trick,

and

and in a Moment what another has gotten by many Years extreme Labour and Pains.

There hath been much Diſcourſe about introducing of Regiſtries into *England*; the Lawyers for the moſt Part object againſt it, alledging that Titles of Land in *England* are ſufficiently ſecure already; wherefore omitting the Conſiderations of ſmall and oblique Reaſons *pro & contra*, it were good that Enquiry were made from the Officers of ſeveral Courts, to what Sum or Value Purchaſers have been damnified for this laſt ten Years, by ſuch fraudulent Conveyances as Regiſtries would have prevented; the tenth Part whereof at a Medium, is the annual Loſs which the People ſuſtain for Want of them, and then Computation is to be made of the annual Charge of regiſtering ſuch extraordinary Conveyances, as would ſecure the Title of Lands; now by comparing theſe two Sums, the Queſtion ſo much agitated may be determined; though ſome think that though few are actually damnified, yet that all are hindered by Fear and deterred from dealing.

Their third Policy is their Bank, the Uſe whereof is to encreaſe Money, or rather to make a ſmall Sum equivalent in Trade to a greater, for the effecting whereof theſe Things are to be conſidered, 1. How much Money will drive the Trade of the Nation. 2. How much current Money there is actually in the Nation. 3. How much Money will ſerve to make all Payments of under 50*l.* or any other more convenient Sum through-

throughout the Year. 4. For what Sum the Keepers of the Bank are unqueſtionable Security: If all theſe four Particulars be well known, then it may alſo be known, how much of the ready Money above-mentioned may ſafely and profitably be lodged in the Bank, and to how much ready current Money the ſaid depoſited Money is equivalent. As for Example, ſuppoſe a Hundred Thouſand Pounds will drive the Trade of the Nation, and ſuppoſe there be but ſixty Thouſand Pounds of ready Money in the ſame; ſuppoſe alſo that Twenty Thouſ. Pounds will drive on and anſwer all Payments made of under 50 *l.* In this Caſe Forty of the Sixty being put into the Bank, will be equivalent to Eighty, which Eighty and Twenty kept out of the Bank do make up an Hundred, (that is to ſay) enough to drive the Trade as was propoſed: Where note that the Bank-keepers muſt be reſponſible for double the Sum intruſted with them, and muſt have Power to levy upon the general, what they happen to loſe unto particular Men.

Upon which Grounds the Bank may freely make uſe of the received Forty Thouſand Pounds, whereby the ſaid Sum, with the like Sum in Credit, makes Eighty Thouſand Pounds, and with the Twenty reſerved an Hundred.

I might here add many more Particulars, but being the ſame as have already been noted by others, I ſhall conclude only with adding one Obſervation which I take to be of Conſequence, *viz.*

viz. That the *Hollanders* do rid their Hands of two Trades, which are of greateſt Turmoil and Danger, and yet of leaſt Profit; the firſt whereof is that of a common and private Soldier, for ſuch they can hire from *England*, *Scotland*, and *Germany*, to venture their Lives for Sixpence a Day, whilſt themſelves ſafely and quietly follow ſuch Trades, whereby the meaneſt of them gain ſix Times as much, and withal by this Entertaining of Strangers for Soldiers, their Country becomes more and more peopled, foraſmuch as the Children of ſuch Strangers are *Hollanders*, and take to Trades, whilſt new Strangers are admitted *ad infinitum*; beſides theſe Soldiers at convenient Intervals, do at leaſt as much Work as is equivalent to what they ſpend, and conſequently by this Way of employing of Strangers for Soldiers, they people the Country, and ſave their own Perſons from Danger and Miſery, without any real Expence, effecting by this Method, what others have in vain attempted by Laws for naturalizing of Strangers, as if Men could be charmed to tranſplant themſelves from their own native into a foreign Country, merely by Words, and for the bare Leave of being called by a new Name. In *Ireland* Laws of Naturalization have had little Effect, to bring in Aliens, and 'tis no Wonder, ſince *Engliſhmen* will not go thither, without they may have the Pay of Soldiers, or ſome other Advantage amounting to Maintenance.

Having

Having intimated the Way by which the *Hollanders* do increaſe their People, I ſhall here digreſs to ſet down the Way of computing the Value of every Head one with another, and that by the Inſtance of People in *England*, *viz.* Suppoſe the People of *England* be Six Millions in Number, that their Expence at 7 *l. per* Head be Forty-two Millions: Suppoſe alſo that the Rent of the Lands be Eight Millions, and the yearly Profit of all the Perſonal Eſtate be Eight Millions more; it muſt needs follow, that the Labour of the People muſt have ſupplied the remaining Twenty-ſix Millions, the which multiplied by Twenty (the Maſs of Mankind being worth twenty Years Purchaſe as well as Land) makes Five hundred and twenty Millions, as the Value of the whole People; which Number divided by ſix Millions, makes above 80 *l.* Sterling to be the Value of each Head of Man, Woman, and Child, and of adult Perſons twice as much; from whence we may learn to compute the Loſs we have ſuſtained by the Plague, by the Slaughter of Men in War, and by the ſending them abroad into the Service of foreign Princes. The other Trade of which the *Hollanders* have rid their Hands, is the old patriarchal Trade of being Cow-keepers, and in a great Meaſure of that which concerns Ploughing and Sowing of Corn, having put that Employment upon the *Danes* and *Polanders*, from whom they have their young Cattle and Corn. Now here we may take Notice, that as Trades

and curious Arts increaſe, ſo the Trade of Huſbandry will decreaſe, or elſe the Wages of Huſbandmen muſt riſe, and conſequently the Rents of Lands muſt fall.

For Proof whereof I dare affirm, that if all the Huſbandmen of *England*, who now earn but 8 *d.* a Day, or thereabouts, could become Tradeſmen and earn 16 *d.* a Day (which is no great Wages, 2 *s.* and 2 *s.* 6 *d.* being uſually given) that then it would be the Advantage of *England* to throw up their Huſbandry, and to make no Uſe of their Lands, but for Graſs Horſes, Milch Cows, Gardens, and Orchards, *&c.* which if it be ſo, and if Trade and Manufacture have increaſed in *England* (that is to ſay) if a greater Part of the People apply themſelves to thoſe Faculties than there did heretofore, and if the Price of Corn be no greater now, than when Huſbandmen were more numerous, and Tradeſmen fewer; it follows from that ſingle Reaſon (though others may be added) that the Rents of Land muſt fall: As for Example, ſuppoſe the Price of Wheat be 5 *s.* or 60 *d.* the Buſhel; now if the Rent of the Land whereon it grows be the third Sheaf, then of the 60 *d.* 20 *d.* is for the Land, and 40 *d.* for the Huſbandman; but if the Huſbandman's Wages ſhould riſe one eighth Part, or from 8 *d.* to 9 *d. per Diem*, then the Huſbandman's Share in the Buſhel of Wheat riſes from 40 *d.* to 45 *d.* And conſequently the Rent of the Land muſt fall from 20 *d.* to 15 *d.* for we ſuppoſe the Price of

the

the Wheat ſtill remains the ſame: Eſpecially ſince we cannot raiſe it, for if we did attempt it, Corn would be brought in to us, (as into *Holland*) from foreign Parts, where the State of Huſbandry was not changed.

And thus I have done with the firſt principal Concluſion, that *A ſmall Territory, and even a few People, may by Situation, Trade, and Policy, be made equivalent to a greater; and that Convenience for Shipping, and Water-carriage, do moſt eminently and fundamentally conduce thereunto.*

CHAP. II.

That ſome Kind of Taxes *and Publick* Levies, *may rather* increaſe *than* diminiſh *the Wealth of the Kingdom.*

IF the Money or other Effects, levied from the People by Way of *Tax*, were deſtroyed and annihilated; then 'tis clear, that ſuch Levies would diminiſh the Commonwealth: Or if the ſame were exported out of the Kingdom without any Return at all, then the Caſe would be alſo the ſame or worſe: But if what is levied as aforeſaid, be only transferred from one Hand to another, then we are only to conſider whether the ſaid Money or Commodities, are taken from an improving Hand, and given to an ill Husband, or *vice verſa:* As for Example, ſuppoſe that Money by Way of Tax, be taken from one who ſpendeth the ſame in ſuperfluous eating and drinking; and delivered to another

who employeth the ſame in improving of Land, in Fiſhing, in working of Mines, in Manufacture, &c. It is manifeſt, that ſuch Tax is an Advantage to the State whereof the ſaid different Perſons are Members: Nay, if Money be taken from him, who ſpendeth the ſame as aforeſaid upon eating and drinking, or any other periſhing Commodity; and the ſame transferr'd to one that beſtoweth it on Cloaths; I ſay, that even in this Caſe, the Commonwealth hath ſome little Advantage; becauſe Cloaths do not altogether periſh ſo ſoon as Meats and Drinks: But if the ſame be ſpent in Furniture of Houſes, the Advantage is yet a little more; if in building of Houſes, yet more; if in improving of Lands, working of Mines, Fiſhing, &c. yet more; but moſt of all, in bringing Gold and Silver into the Country; becauſe thoſe Things are not only not periſhable, but are eſteemed for Wealth at all Times, and every where: Whereas other Commodities which are periſhable, or whoſe Value depends upon the Faſhion, or which are contingently ſcarce and plentiful, are Wealth but *pro hic & nunc*, as ſhall be elſewhere ſaid.

In the next Place if the People of any Country, who have not already a full Employment, ſhould be enjoined or taxed to work upon ſuch Commodities as are imported from abroad; I ſay that ſuch a Tax alſo doth improve the Commonwealth.

Moreover,

Moreover, if Perſons who live by begging, cheating, ſtealing, gaming, borrowing without Intention of reſtoring; who by thoſe Ways do get from the credulous and careleſs, more than is ſufficient for the Subſiſtence of ſuch Perſons; I ſay, that although the State ſhould have no preſent Employment for ſuch Perſons, and conſequently ſhould be forced to bear the whole Charge of their Livelihood; yet it were more for the publick Profit to give all ſuch Perſons a regular and competent Allowance by publick Tax, than to ſuffer them to ſpend extravagantly, at the only Charge of credulous, careleſs, and good-natured People; and to expoſe the Commonwealth to the Loſs of ſo many able Men, whoſe Lives are taken away for the Crimes which ill Diſcipline doth occaſion.

On the contrary, if the Stocks of laborious and ingenious Men, who are not only beautifying the Country where they live, by elegant Dyet, Apparel, Furniture, Houſing, pleaſant Gardens, Orchards, and publick Edifices, *&c.* but are alſo increaſing the Gold, Silver, and Jewels of the Country by Trade and Arms; I ſay, if the Stock of theſe Men ſhould be diminiſhed by a Tax, and transferred to ſuch as do nothing at all but eat and drink, ſing, play, and dance; nay to ſuch as ſtudy the Metaphyſicks, or other needleſs Speculation; or elſe employ themſelves in any other Way, which produces no material Thing, or Things of real Uſe and Value in the Commonwealth: In this Caſe,

the Wealth of the Publick will be diminiſhed, otherwiſe than as ſuch Exerciſes are Recreations and Refreſhments of the Mind; and which being moderately uſed, do qualify and diſpoſe Men to what in itſelf is more conſiderable.

Wherefore upon the whole Matter, to know whether a Tax will do Good or Harm, the State of the People, and their Employments, muſt be well known; (that is to ſay) what Part of the People are unfit for Labour by their Infancy or Impotency; and alſo what Part are exempt from the ſame, by Reaſon of their Wealth, Function, or Dignities; or by Reaſon of their Charge and Employments; otherwiſe than in governing, directing, and preſerving thoſe who are appointed to Labour and Arts.

2. In the next Place Computation muſt be made, what Part of thoſe who are fit for Labour and Arts as aforeſaid, are able to perform the Work of the Nation in its preſent State and Meaſure.

3. It is to be conſidered, whether the Remainder can make all or any Part of thoſe Commodities which are imported from abroad; which of them, and how much in particular: The Remainder of which Sort of People (if any be) may ſafely and without poſſible Prejudice to the Commonwealth, be employed in Arts and Exerciſes of Pleaſure and Ornament; the greateſt whereof is the Improvement of natural Knowledge,

Having

Having thus in general illuſtrated this Point, which I think needs no other Proof but Illuſtration, I come next to intimate that no Part of *Europe* hath paid ſo much by Way of Tax, and publick Contribution, as *Holland* and *Zealand* for this laſt 100 Years; and yet no Country hath in the ſame Time increaſed their Wealth comparably to them: And it is manifeſt they have followed the general Conſiderations abovementioned; for they tax Meats and Drinks moſt heavily of all, to reſtrain the exceſſive Expence of thoſe Things, which 24 Hours doth (as to the Uſe of Man,) wholly annihilate, and they are more favourable to Commodities of greater Duration.

Nor do they tax according to what Men gain, but in extraordinary Caſes, but always according to what Men ſpend: And moſt of all, according to what they ſpend needleſly, and without Proſpect of Return. Upon which Grounds, their Cuſtoms upon Goods imported and exported are generally low; as if they intended by them, only to keep an Account of their foreign Trade, and to retaliate upon their Neighbour States the Prejudices done them by their Prohibitions and Impoſitions.

It is further to be obſerved, that ſince the Year 1636, the Taxes and publick Levies made in *England*, *Scotland*, and *Ireland*, have been prodigiouſly greater than at any Time heretofore; and yet the ſaid Kingdoms have increaſed in their Wealth and Strength, for theſe laſt Forty Years, as ſhall hereafter be ſhewn.

It is ſaid that the King of *France*, at preſent, doth levy the fifth Part of his People's Wealth; and yet great Oſtentation is made of the preſent Riches and Strength of that Kingdom. Now great Care muſt had in diſtinguiſhing between the Wealth of the People, and that of an abſolute Monarch, who taketh from the People, where, when, and in what Proportion he pleaſeth. Moreover, the Subjects of two Monarchs may be equally rich, and yet one Monarch may be double as rich as the other; *viz.* If one take the tenth Part of the People's Subſtance to his own Diſpoſe, and the other but the twentieth; nay the Monarch of a poorer People may appear more ſplendid and glorious than that of a richer, which perhaps may be ſomewhat the Caſe of *France*, as hereafter ſhall be examined. As an Inſtance and Application of what hath been ſaid, I conceive that in *Ireland* wherein are about 1200 Thouſand People, and near 300 Thouſand Smokes or Hearths, it were more tolerable for the People, and more profitable for the King, that each Head paid 2 *s.* worth of Flax, than that each Smoke ſhould pay 2 *s.* in Silver; and that for the following Reaſons.

1. *Ireland* being under-peopled, and Land and Cattle being very cheap, there being every where Store of Fiſh and Fowl; the Ground yielding excellent Roots (and particularly that bread-like Root Potatoes) and withal they being able to perform their Husbandry, with ſuch Harneſs and

and Tackling as each Man can make with his own Hands, and living in ſuch Houſes as almoſt every Man can build; and every Houſe-wife being a Spinner and Dyer of Wool and Yarn, they can live and ſubſiſt after their preſent Faſhion, without the Uſe of Gold or Silver Money; and can ſupply themſelves with the Neceſſaries above-named, without labouring 2 Hours *per Diem:* Now it hath been found, that by Reaſon of Inſolvencies ariſing rather from the Uſeleſneſs than Want of Money among theſe poor People, that from 300 Thouſand Hearths, which ſhould have yielded 30 Thouſand Pound *per Annum*, not 15 Thouſand Pound of Money could be levied: Whereas it is eaſily imagined, that four or five People dwelling in that Cottage which hath but one Smoke, could eaſily have planted a Ground-plot of about 40 Foot ſquare with Flax, or the 50 Part of an Acre, for ſo much Ground will bear eight or ten Shillings worth of that Commodity; and the Rent of ſo much Ground in few Places amounts to a Penny *per Annum.* Nor is there any Skill requiſite to this Practice, wherewith the Country is not already familiar. Now as for a Market for the Flax, there is imported into *Holland* itſelf, over and above what that Country produces, as much Flax as is there ſold for between Eightſcore and Two Hundred Thouſand Pound; and into *England* and *Ireland* is imported as much Linnen Cloth made of Flax, and there ſpent, as is worth above $\frac{1}{2}$ a Million

Million of Money. As ſhall hereafter be ſhewn.

Wherefore having ſhewn, that Silver Money is uſeleſs to the poor People of *Ireland*; that half the Hearth Money could not by raiſed by Reaſon thereof; that the People are not a fifth Part employed; that the People and Land of *Ireland* are competently qualified for Flax; that one Penny-worth of Land will produce ten Shillings worth of the ſame; and that there is Market enough and enough for above 100000 Pounds worth; I conceive my Propoſition ſufficiently proved; at leaſt to ſet forwards and promote a Practice which both the preſent Law and Intereſt of the Country doth require: Eſpecially, ſince if all the Flax ſo produced ſhould yield nothing, yet there is nothing loſt; the ſame Time having been worſe ſpent before. Upon the ſame Grounds, the like Tax of 2 *s. per* Head, may be raiſed with the like Advantage upon the People of *England*, which will amount to Six Hundred Thouſand Pound *per Annum*; to be paid in Flax, manufactured into all the Sorts of Linnens, Threads, Tapes, and Laces; which we now receive from *France*, *Flanders*, *Holland*, and *Germany*; the Value whereof doth far exceed the Sum laſt-mentioned, as hath appeared by the Examination of Particulars.

It is obſerved by Clothiers, and others, who employ great Numbers of poor People, that when Corn is extremely plentiful, that the Labour of the Poor is proportionably dear, and

ſcarce

ſcarce to be had at all (ſo licentious are they who labour only to eat, or rather to drink.) Wherefore when ſo many Acres ſown with Corn, as do uſually produce a ſufficient Store for the Nation, ſhall produce perhaps double to what is expected or neceſſary; it ſeems not unreaſonable that this common Bleſſing of God ſhould be applied to the common Good of all People, repreſented by their Sovereign; much rather than the ſame ſhould be abuſed by the vile and brutiſh Part of Mankind, to the Prejudice of the Commonwealth: And conſequently, that ſuch Surpluſage of Corn ſhould be ſent to publick Store-houſes; from thence to be diſpoſed of, to the beſt Advantage of the Publick.

Now if the Corn ſpent in *England*, at five Shillings *per* Buſhel Wheat, and two Shillings Sixpence Barley, be worth ten Millions *communibus annis*; it follows that in Years of great Plenty, when the ſaid Grains are one third Part cheaper, that a vaſt Advantage might accrue to the Commonwealth, which now is ſpent in over-feeding of the People, in Quantity or Quality; and ſo indiſpoſing them to their uſual Labour.

The like may be ſaid of Sugar, Tobacco, and Pepper, which Cuſtom hath now made neceſſary to all Sorts of People; and which the over-planting of them hath made unreaſonably cheap: I ſay it is not abſurd that the Publick ſhould be advantaged by this extraordinary Plenty. That

That an Exciſe ſhould be laid on Currants alſo, is not unreaſonable; not only for this but for other Reaſons alſo.

The Way of the preſent Militia or Trained Bands is a gentle Tax upon the Country; becauſe it is only a few Days Labour in the Year, of a few Men in reſpect of the Whole; uſing their own Goods, that is their own Arms. Now if there be three Millions of Males in *England*, there muſt be above 200 Thouſand of them who are between the Age of 16 and 30, unmarried Perſons, and who live by their Labour and Service; for of ſo many, or thereabouts, the preſent Militia conſiſts.

Now if an hundred and fifty Thouſand of theſe were armed and trained as Foot, and fifty Thouſand as Horſe, (Horſe being of ſpecial Advantage in Iſlands) the ſaid Forces at Land, with thirty Thouſand Men at Sea, would, by God's ordinary Bleſſing, defend this Nation, being an Iſland, againſt any Force in View: But the Charge of arming, diſciplining, and rendezvouſing all theſe Men twice or thrice a Year, would be a very gentle Tax, levied by the People themſelves, and paid to themſelves. Moreover if out of the ſaid Number ; Part were ſelected of ſuch as are more than ordinarily fit and diſpoſed for War, and to be exerciſed and rendezvouſed fourteen or fifteen Times *per Annum*; the Charge thereof being but a Fortnight's Pay in the Year, would be alſo a very gentle Tax.

Laſtly,

Laſtly, if out of this laſt-mentioned Number, again ſhould be ſelected, making about ſixteen thouſand Foot, and near ſix thouſand Horſe to be exerciſed and rendezvouſed forty Days in the Year; I ſay that the Charge of all theſe three Militias, allowing the latter ſix Weeks Pay *per Annum* would not coſt above one hundred and twenty thouſand Pound *per Annum*; which I take to be an eaſy Burthen for ſo great a Benefit.

Foraſmuch as the preſent Navy of *England* requires thirty-ſix thouſand Men to man it; and for that the *Engliſh* Trade of Shipping requires about forty-eight thouſand Men to manage it alſo; it follows, that to perform both well, there ought to be about ſeventy-two thouſand Men, (and not eighty-four thouſand) competently qualified for theſe Services: For Want whereof we ſee, that it is a long while before a Royal Navy can be manned; which till it be, is of no effectual Uſe, but lies at Charge. And we ſee likewiſe upon theſe Occaſions, that Merchants are put to great Straights and Inconveniencies; and do pay exceſſive Rates for the carrying on their Trade. Now if twenty-four thouſand able-bodied Tradeſmen, were by ſix thouſand of them *per Annum*, brought up and fitted for Sea-Service; and for their Encouragement allowed 20 *s. per Annum* for every Year they had been at Sea, even when they ſtay at home, not exceeding 6 *l.* for thoſe who have ſerved ſix Years or upward; it follows, that about

bout 72000 *l.* at the Medium of 3 *l. per* Man, would pay the whole Number of twenty-four Thouſand; and ſo, foraſmuch as half the Seamen which manage the Merchants Trade, are ſuppoſed to be always in Harbour, and are about twenty-four thouſand Men, with the ſaid half together of the Auxiliaries laſt-mentioned, would upon all Emergencies man out the whole Royal Navy with thirty-ſix Thouſand, and leaving to the Merchants twelve Thouſand of the abler Auxiliaries, to perform their Buſineſs in Harbour, till others come home from Sea; and thus thirty-ſix Thouſand, twenty-four Thouſand, and twelve Thouſand, make the ſeventy-two Thouſand abovementioned: I ſay that more than this Sum of 72000 *l.* is fruitleſly ſpent, and over-paid by the Merchants, whenſoever a great Fleet is to be fitted out. Now theſe whom I call Auxiliary Seamen, are ſuch as have another Trade beſides, wherewith to maintain themſelves, when they are not employed at Sea; and the Charge of maintaining them, though 72000 *l. per Annum*, I take to be little or nothing, for the Reaſons above-mentioned, and conſequently an eaſy Tax to the People, becauſe levied by, and paid to themſelves.

As we propounded that *Ireland* ſhould be taxed with Flax, and *England* by Linnen, and other Manufacture of the ſame, I conceive that *Scotland* alſo might be taxed as much, to be paid in Herrings, as *Ireland* in Flax: Now the three

three Taxes (*viz.*) of Flax, Linnen, and Herrings, and the Maintenance of the triple Militia, and of the auxiliary Seamen above-mentioned, do all five of them together, amount to one Million of Money, the raiſing whereof is not a Million ſpent, but Gain unto the Commonwealth, unleſs it can be made appear, that by Reaſon of all, or any of them, the Exportation of Woollen Manufactures, Lead, and Tin, are leſſened; or of ſuch Commodities as our own *Eaſt* and *Weſt India* Trade do produce, foraſmuch as I conceive, that the Exportation of theſe laſt-mentioned Commodities is the Touch-ſtone whereby the Wealth of *England* is tried, and the Pulſe whereby the Health of the Kingdom may be diſcerned.

CHAP. III.

That France *cannot, by Reaſon of natural and perpetual Impediments, be more powerful at Sea, than the* Engliſh, *or* Hollanders, *now are, or may be.*

POWER at Sea conſiſts chiefly of *Men able to fight at Sea*, and that in ſuch Shipping as is moſt proper for the Seas wherein they ſerve; and thoſe are, in theſe Northern Seas, Ships from between three Hundred to one Thouſand three hundred Tons; and of thoſe ſuch as draw much Water, and have a deep Latch in the Sea, in order to keep a good Wind, and not to fall to Leeward, a matter of vaſt Advantage

vantage in Sea Service: Wherefore it is to be examined, 1. Whether the King of *France* hath Ports in the Northern Seas (where he hath moſt Occaſion for his Fleets of War, in any Conteſts with *England*) able to receive the Veſſels above-mentioned, in all Weathers, both in Winter and Summer Seaſon. For if the King of *France* ſhould bring to Sea an equal Number of fighting Men with the *Engliſh* and *Hollanders*, in ſmall floaty Leeward Veſſels, he would certainly be of the weaker Side. For a Veſſel of one thouſand Tons manned with five hundred Men, fighting with five Veſſels of two hundred Tons, each manned with one hundred Men a piece, ſhall in common Reaſon have the better offenſively and defenſively; foraſmuch as the great Ship can carry ſuch Ordnance, as can reach the ſmall ones at a far greater Diſtance than thoſe can reach, or at leaſt hurt the other; and can batter, and ſink at a Diſtance, when ſmall ones can ſcarce pierce.

Moreover it is more difficult for Men out of a ſmall Veſſel to enter a tall Ship, than for Men from a higher Place to leap down into a lower; nor is ſmall Shot ſo effectual upon a tall Ship, as *vice verſa*.

And as for Veſſels drawing much Water, and conſequently keeping a good Wind, they can take or leave leeward Veſſels at Pleaſure, and ſecure themſelves from being boarded by them: Moreover the windward Ship has a fairer Mark at a leeward Ship, than *vice verſa*; and can

place

place her Shot upon ſuch Parts of the leeward Veſſel, as upon the next Tack will be under Water.

Now then the King of *France* having no Ports able to receive large windward Veſſels, between *Dunkirk* and *Uſhant*, what other Ships he can bring into thoſe Seas, will not be conſiderable. As for the wide Ocean, which his Harbours of *Breſt* and *Charente* do look into, it affordeth him no Advantage upon an Enemy; there being ſo great a Latitude of engaging or not, even when the Parties are in Sight of each other.

Wherefore, although the King of *France* were immenſely rich, and could build what Ships he pleaſed, both for Number and Quality; yet if he have not Ports to receive and ſhelter that Sort and Size of Shipping, which is fit for his Purpoſe; the ſaid Riches will in this Caſe be fruitleſs, and a mere Expence without any Return or Profit. Some will ſay that other Nations cannot build ſo good Ships as the *Engliſh*; I indeed hope they cannot; but becauſe it ſeems too poſſible that they may ſooner or later, by Practice and Experience, I ſhall not make uſe of that Argument, having bound myſelf to ſhew, that the Impediments of *France*, (as to this Purpoſe) are natural and perpetual. Ships and Guns do not fight of themſelves, but Men who act and manage them; wherefore it is more material to ſhew, that the King of *France* neither hath, nor can have Men ſufficient to

man a Fleet of equal Strength to that of the King of *England*, (*viz.*)

The King of *England*'s Navy, conſiſts of about 70 thouſand Tons of Shipping, which requires 36 thouſand Men to man it; theſe Men being ſuppoſed to be divided into eight Parts, I conceive that one eighth Part muſt be Perſons of great Experience and Reputation in Sea-Service: another eighth Part muſt be ſuch as have uſed the Sea ſeven Years and upwards; half of them, or $\frac{4}{8}$ Parts more muſt be ſuch as have uſed the Sea above a Twelvemonth, *viz.* two, three, four, five, or ſix Years, allowing but one Quarter of the whole Complements to be ſuch as never were at Sea at all, or at moſt but one Voyage, or upon one Expedition; ſo that at a Medium I reckon, that the whole Fleet muſt be Men of three or four Years Growth, one with another. *Fournier*, a late judicious Writer, making it his Buſineſs to perſuade the World, how conſiderable the King of *France* was, or might be, at Sea, in the 92*d* and 93*d*. Pages of his *Hydrography*, ſaith, That there was one Place in *Britany*, which had furniſhed the King with 1400 Seamen, and that perhaps the whole Sea-Coaſt of *France* might have furniſhed him with 15 Times as many: Now ſuppoſing his whole Allegation were true, yet the ſaid Number amounts but to 21000; all which, if the whole Trade of Shipping in *France* were quite and clean abandoned, would not, by above $\frac{1}{3}$, man out a Fleet equivalent to that of the King of *Eng-*

England: And if the Trade were but barely kept alive, there would not be ⅓ Part Men enough to man the ſaid Fleet.

But if the Shipping Trade of *France* be not above ¼ as great as that of *England*, and that ⅓ Part of the ſame, namely the Fiſhing Trade to the Banks of *Newfoundland*, is not peculiar, nor fixt to the *French*; then I ſay that if the King of *England* (having Power to preſs Men) cannot under two or three Months Time man his Fleet, then the King of *France*, with leſs than ¼ of the ſame Help, can never do it at all; for in *France* (as ſhall elſewhere be ſhewn) there are not above 150000 Ton of trading Veſſels, and conſequently not above 15000 Seamen, reckoning a Man to every ten Ton. As it has been ſhewn that the King of *France* cannot at preſent man ſuch a Fleet as is above deſcribed, we come next to ſhew that he never can, being under natural and perpetual Impediments, *viz.*

1. If there be but 15000 Seamen in all *France*, to manage its Trade, it is not to be ſuppoſed, that the ſaid Trade ſhould be extinguiſhed, nor that it ſhould ſpare above 5 of the ſaid 15000 towards manning the Fleet which requires 35 Thouſand.

Now the deficient 30000 muſt be ſupplied one of theſe four Ways; either, firſt, by taking in Landmen, of which Sort there muſt not be above 10000, ſince the Seamen will never be contented without being the major Part, nor do they heartily wiſh well to Landmen at all, or

rejoice even at thoſe Succeſſes of which the Landmen can claim any Share; thinking it hard that themſelves, who are bred to miſerable, painful, and dangerous Employments, (and yet profitable to the Commonwealth) ſhould, at a Time when Booty and Purchaſe is to be gotten, be clogged or hindered by any Conjunction with Landmen, or forced to admit thoſe to an equal Share with themſelves. 2. The Seamen, which we ſuppoſe 20000, muſt be had, that is hired, from other Nations, which cannot be without tempting them with ſo much Wages, as exceeds what is given by Merchants, and withal to counterpoiſe the Danger of being hanged by their own Prince, and allowed no Quarter if they are taken; the Trouble of conveying themſelves away, when Reſtraints and Prohibitions are upon them; and alſo the Infamy of having been Apoſtates to their own Country and Cauſe: I ſay their Wages muſt be more than double to what their own Prince gives them, and their Aſſurance muſt be very great, that they ſhall not be at long Run abuſed or ſlighted by thoſe who employed them; (as hating the Traitor although they love the Treaſon.) I ſay moreover, that thoſe who will be thus tempted away, muſt be of the baſeſt and lewdeſt Sort of Seamen, and ſuch as have not enough of Honour and Conſcience to qualify them for any Truſt, or gallant Performance. 3. Another Way to increaſe Seamen, is to put great Numbers of Landmen upon Ships of War, in order to their being

being Seamen; but this Courſe cannot be effectual, not only for the above-mentioned Antipathy between Landmen and Seamen, but alſo becauſe it is ſeen that Men at Sea do not apply themſelves to Labour and Practice, without more Neceſſity than happens in over-manned Shipping. For where there are 50 Men in a Veſſel, that 10 can ſufficiently navigate, the ſupernumerary 40 will improve little: But where there ſhall be of 10 but 1 or 2 Supernumeraries, there Neceſſity will often call upon every Man to ſet his Hand to the Work, which muſt be well done at the Peril of their own Lives. Moreover Seamen ſhifting Veſſels almoſt every 6 or 12 Months, do ſometimes ſail in ſmall Barks, ſometimes in midling Ships, and ſometimes in great Veſſels of Defence; ſometimes in Lighters, ſometimes in Hoighs, ſometimes in Ketches, ſometimes in three maſted Ships, ſometimes they go to the Southward, ſometimes to the Northward, ſometimes the Coaſt, ſometimes they coaſt the Ocean; by all which Variety of Service, they do in Time compleat themſelves in every Part and Circumſtance of their Faculty: Whereas thoſe who go out for a Summer, in a Man of War, have not that Variety of Practice, nor a direct Neceſſity of doing any Thing at all.

Beſides it is 3 or 4 Years at a Medium, wherein a Seaman can be made; neither can there be leſs than three Seamen to make a fourth of a Landman: Conſequently the 15000 Seamen of *France* can increaſe but 5000 Seamen

3 or 4 Years, and unleſs their Trade ſhould increaſe with their Seamen in Proportion, the King muſt be forced to bear the Charge of this Improvement out of the publick Stock, which is intolerable. So as the Queſtion which now remains is, whether the Shipping Trade of *France* is like to increaſe? Upon which Account it is to be conſidered, 1. That *France* is ſufficiently ſtored with all Kind of Neceſſaries within itſelf; as with Corn, Cattle, Wine, Salt, Linnen Cloth, Paper, Silk, Fruits, *&c.* So as they need little Shipping to import more Commodities of Weight or Bulk; neither is there any Thing of Bulk exported out of *France*, but Wines and Salt; the Weight whereof is under 100,000 Tun *per Annum*, yielding not Employment to above 25000 Ton of Shipping, and theſe are for the moſt Part *Dutch* and *Engliſh*, who are not only already in Poſſeſſion of the ſaid Trade, but alſo are better fitted to maintain it than the *French* are, or perhaps ever can be: And that for the following Reaſons, (*viz.*) 1. Becauſe the *French* cannot victual ſo cheap as the *Engliſh* and *Dutch*, nor ſail with ſo few Hands. 2. The *French* for Want of good Coaſts and Harbours, cannot keep their Ships in Port, under double the Charge that the *Engliſh* and *Hollanders* can. 3. By Reaſon of Paucity, and Diſtance of their Ports, one from another, their Seamen and Tradeſmen relating to Shipping, cannot correſpond with, and aſſiſt, one another ſo eaſily, cheaply, and advantageouſly, as in other

ther Places. Wherefore if their Shipping Trade is not likely to increaſe within themſelves, and much leſs to increaſe by their beating out the *Engliſh* and *Hollanders* from being the Carriers of the World; it follows that their Seamen will not be increaſed by the Increaſe of their ſaid Trade: Wherefore, and for that they are not like to be increaſed by any of the ſeveral Ways above-ſpecified, and for that their Ports are not fit to receive Ships of Burthen and Quality fit for their Purpoſe; and that by Reaſon of the leſs Fitneſs of their Ports than that of their Neighbours; I conceive that what was propounded hath been competently proved.

The afore-named *Fournier* in the 92*d* and 93*d* Pages of his *Hydrography* hath laboured to prove the contrary of all this, unto which I refer the Reader: Not thinking his Arguments of any Weight at all in the preſent Caſe. Nor indeed doth he make his Compariſons with the *Engliſh* or *Hollanders*, but with the *Spaniards*, who, nor the Grand Seignior, (the latter of whom hath greater Advantages to be powerful at Sea than the King of *France*) could ever attain to any illuſtrious Greatneſs in Naval Power: Having often attempted, but never ſucceeded in the ſame. Nor is it eaſy to believe, that the King of *England* ſhould, for ſo many Years, have continued his Title to the Sovereignty of the *Narrow Seas* againſt his Neighbours (ambitious enough to have gotten it from him) had not their Impediments been natural and perpetual, and ſuch, as we ſay, do obſtruct the King of *France*.

CHAP. IV.

That the People and Territories of the King of England, *are naturally near as conſiderable for Wealth and Strength, as thoſe of* France.

THE Author of the State of *England*, among the many uſeful Truths and Obſervations he hath ſet down, delivers the Proportion between the Territories of *England* and *France* to be as 30 to 82; the which if it be true, then *England*, *Scotland*, and *Ireland*, with the Iſlands unto them belonging, will, taken all together, be near as big as *France*. Though I ought to take all Advantages for proving the Paradox in Hand; yet I had rather grant that *England*, *Scotland*, and *Ireland*, with the Iſlands before-mentioned; together with the planted Parts of *Newfoundland*, *New-England*, *New-Netherland*, *Virginia*, *Mary-Land*, *Carolina*, *Jamaica*, *Bermudas*, *Barbadoes*, and all the reſt of the *Carribbee* Iſlands, with what the King hath in *Aſia* and *Africa*, do not contain ſo much Territory as *France*, and what planted Land the King of *France* hath alſo in *America*. And if any Man will be heterodox in Behalf of the *French* Intereſt, I would be contented againſt my Knowledge and Judgment to allow the King of *France*'s Territories to be a ſeventh, ſixth, or even a fifth greater than thoſe of the King of *England*;

England; believing that both Princes have more Land, than they do employ to its utmoſt Uſe.

And here I beg Leave, (among the ſeveral Matters which I intend for ſerious,) to interpoſe a jocular, and perhaps whimſical, Digreſſion, and which I indeed deſire Men to look upon rather as a Dream, or Reſvery, than a rational Propoſition; the which is, that if all the Moveables and People of *Ireland*, and of the Highlands of *Scotland*, were tranſported into the reſt of *Great Britain*; that then the King and his Subjects, would thereby become more rich and ſtrong, both offenſively and defenſively, than now they are.

'Tis true, I have heard many wiſe Men ſay, when they were bewailing the vaſt Loſſes of the *Engliſh*, in preventing and ſuppreſſing Rebellions in *Ireland*, and conſidering how little Profit hath returned either to the King or Subjects of *England*, for their 500 Years doing and ſuffering in that Country; I ſay, I have heard wiſe Men (in ſuch their Melancholies) wiſh, that (the People of *Ireland* being ſaved) that Iſland were ſunk under Water: Now it troubles me, that the Diſtemper of my own Mind in this Point, carries me to dream that the Benefit of thoſe Wiſhes may practically be obtained, without ſinking that vaſt mountainous Iſland under Water, which I take to be ſomewhat difficult; for although *Dutch* Engineers may drain its Bogs, yet I know no Artiſts that could ſink its Mountains. If ingenious and learned Men

Men (among whom I reckon Sir *Tho. More*, and *Des Cartes*) have diſputed, that we who think ourſelves awake, are or may be really in a Dream; and ſince the greateſt Abſurdities of Dreams are but a prepoſterous and tumultuary Contexture of Realities; I will crave the Umbrage of theſe great Men laſt named, to ſay ſomething for this wild Conception, with Submiſſion to the better Judgment of all thoſe that can prove themſelves awake.

If there were but one Man living in *England*, then the Benefit of the whole Territory could be but the Livelihood of that one Man: But if another Man were added, the Rent or Benefit of the ſame would be double, if two, triple; and ſo forward, until ſo many Men were planted in it, as the whole Territory could afford Food unto: For if a Man would know what any Land is worth, the true and natural Queſtion muſt be, How many Men will it feed? How many Men are there to be fed? But to ſpeak more practically, Land of the ſame Quantity and Quality in *England*, is generally worth 4 or 5 Times as much as in *Ireland*, and but $\frac{1}{4}$ or $\frac{1}{3}$ of what it is worth in *Holland*; becauſe *England* is 4 or 5 Times better peopled than *Ireland*, and but $\frac{1}{4}$ ſo well as *Holland*. And moreover, where the Rent of Land is advanced by Reaſon of Multitude of People, there the Number of Years Purchaſe, for which the Inheritance may be ſold, is alſo advanced, though perhaps not in the very ſame Proportion; for 20 *s. per Annum* in *Ireland*,

Ireland, may be worth but 8 *l.* and in *England*, where Titles are very ſure, above 20 *l.* in *Holland* above 30 *l.*

I ſuppoſe that in *Ireland*, and the Highlands in *Scotland*, there may be about 1800000 People, or about $\frac{1}{5}$ Part of what is in all the 3 Kingdoms: Wherefore the firſt Queſtion will be, whether *England*, *Wales*, and the Lowlands of *Scotland*, cannot afford Food, (that is to ſay) Corn, Fiſh, Fleſh, and Fowl, to $\frac{1}{5}$ Part more People than are at preſent planted upon it, with the ſame Labour that the ſaid $\frac{1}{5}$ Part do now take where they are? For if ſo, then what is propounded is naturally poſſible. 2. It is to be enquired, what the Value of the Immoveables (which upon ſuch Removal muſt be left behind) are worth? For if they be worth leſs than the Advancement of the Price of Land in *England* will amount unto; then the Propoſal is to be conſidered. 3. If the deſerted Lands, and the Immoveables left behind upon them, may be ſold for Money, or if no other Nation ſhall dare meddle with them, without paying well for them, and if the Nation who ſhall be admitted, ſhall be leſs able to prejudice and annoy the Tranſplantees into *England* than before; then I conceive that the whole Propoſal will be a pleaſant and a profitable Dream indeed.

As to the firſt Point, whether *England* and the Lowlands of *Scotland* can maintain $\frac{1}{5}$ Part more People than they now do (that is to ſay) 9 Millions of Souls in all? For Anſwer thereunto,

unto, I firſt ſay, that the ſaid Territories of *England*, and the Lowlands of *Scotland*, contain about 36 Millions of Acres, that is 4 Acres for every Head, Man, Woman, and Child; but the United Provinces do not allow above 1 Acre and ½, and *England* itſelf, reſcinding *Wales*, hath but 3 Acres to every Head, according to the preſent State of Tillage and Husbandry. Now if we conſider that *England* having but 3 Acres to a Head as aforeſaid, doth ſo abound in Victuals, as that it maketh Laws againſt the Importation of Cattle, Fleſh, and Fiſh from abroad; and that the draining of Fens, improving of Forreſts, incloſing of Commons, Sowing of St. Foyne and Clover-graſs, be grumbled againſt by Landlords, as the Way to depreſs the Price of Victuals; then it plainly follows, that leſs than 3 Acres improved as it may be, will ſerve the Turn, and conſequently that 4 will ſuffice abundantly. I could here ſet down the very Number of Acres that would bear Bread and Drink, Corn, together with Fleſh, Butter, and Cheeſe, ſufficient to victual 9 Millions of Perſons, as they are victualled in Ships and regular Families; but ſhall only ſay in general, that 12 Millions of Acres, *viz.* ⅓ of 36 Millions, will do it, ſuppoſing that Roots, Fruits, Fowl, and Fiſh. and the ordinary Profit of Lead, Tin, Iron-Mines, and Woods, would piece up any Defect that may be feared.

As to the ſecond, I ſay, that the Land and Houſing in *Ireland*, and the Highlands of *Scotland*

land, at the preſent Market Rates, are not worth 13 Millions of Money; nor would the actual Charge of making the Tranſplantation propoſed amount to 4 Millions more: So then the Queſtion will be, whether the Benefit expected from this Tranſplantation will exceed 17 Millions.

To which I ſay, that the Advantage will probably be near 4 Times the laſt-mentioned Sum, or about 69300000 *l.* For if the Rent of all *England* and *Wales*, and the Lowlands of *Scotland*, be about 9 Millions *per Annum*, and if the fifth Part of the People be ſuperadded unto the preſent Inhabitants of thoſe Countries; then the Rent will amount unto 10,008,000 *l.* and the Number of Years Purchaſe will riſe from $17\frac{1}{2}$, to $\frac{1}{5}$ more, which is 21. So as the Land which is now worth but 9 Millions *per Annum*, at $17\frac{1}{2}$ Years Purchaſe, making 157 Millions and $\frac{1}{2}$, will then be worth 10800000 *l.* at 21 Years Purchaſe; *viz.* 226,800000 *l.* that is 69,300000 *l.* more than it was before.

And if any Prince, willing to enlarge his Territories, will give any Thing more than $6\frac{1}{2}$ Millions, or half the preſent Value for the ſaid relinquiſhed Lands, which are eſtimated to be worth 13 Millions; then the whole Profit will be above 75,800,600 *l.* or above 4 Times the Loſs, as the ſame was above computed. But if any Man ſhall object, that it will be dangerous unto *England*, that *Ireland* ſhould be in the Hands of any other Nation; I anſwer in ſhort, that that Nation, whoever ſhall purchaſe it (being

ing divided by Means of the ſaid Purchaſe) ſhall not be more able to annoy *England* than now in its united Condition. Nor is *Ireland* nearer *England*, than *France* and *Flanders*.

Now if any Man ſhall deſire a more clear Explanation, how, and by what Means, the Rents of Lands ſhall riſe by this cloſer Cohabitation of People above deſcribed? I anſwer, that the Advantage will ariſe in tranſplanting about 1800000 People from the poor and miſerable Trade of Huſbandry, to more beneficial Handicrafts: For when the Superaddition is made, a very little Addition of Husbandry to the ſame Lands will produce $\frac{1}{5}$ more of Food, and conſequently the additional Hands, earning but 40*s. per Annum* (as they may very well do, nay to 8 *l. per Annum*) at ſome other Trade; the Superlucration will be above 3,600000 *l. per Annum*, which at 20 Years Purchaſe is 70 Millions. Moreover, as the Inhabitants of Cities and Towns ſpend more Commodities, and make greater Conſumptions, than thoſe who live in wild, thin-peopled Countries; ſo when *England* ſhall be thicker peopled, in the ſame Manner before deſcribed, the very ſame People ſhall then ſpend more, than when they lived more ſordidly and inurbanely, and farther aſunder, and more out of the Sight, Obſervation, and Emulation of each other; every Man deſiring to put on better Apparel when he appears in Company, than when he has no Occaſion to be ſeen.

I fur-

I further add, that the Charge of the Government, civil, military, and eccleſiaſtical, would be more cheap, ſafe and effectual in this Condition of cloſer Cohabitation than otherwiſe; as not only Reaſon, but the Example of the United Provinces doth demonſtrate.

But to let this whole Digreſſion paſs for a mere Dream, I ſuppoſe 'twill ſerve to prove, that in Caſe the King of *England*'s Territories ſhould be a little leſs than thoſe of the King of *France*, that foraſmuch as neither of them are over-peopled, that the Difference is not material to the Queſtion in Hand; wherefore ſuppoſing the King of *France*'s Advantages to be little or nothing in this Point of Territory, we come next to examine and compare the Number of Subjects which each of theſe Monarchs doth govern.

The Book called the State of *France*, maketh that Kingdom to conſiſt of 27000 Pariſhes; and another Book written by a ſubſtantial Author, who profeſſedly inquires into the State of the Church and Churchmen of *France*, ſets it down as an extraordinary Caſe, that a Pariſh in *France* ſhould have 600 Souls; wherefore I ſuppoſe that the ſaid Author (who hath ſo well examined the Matter) is not of Opinion that every Pariſh, one with another, hath above 500; by which Reckoning the whole People of *France* are about 13 Millions and $\frac{1}{2}$; now the People of *England*, *Scotland*, and *Ireland*, with the Iſlands adjoining, by Computation from the Numbers

of

of Pariſhes, which commonly have more People in Proteſtant Churches than in Popiſh Countries; as alſo from the Hearth-money, Poll-money, and Exciſe, do amount to about 9 Millions and $\frac{1}{2}$.

There are in *New-England* about 16000 Men muſtered in Arms; about 24000 able to bear Arms, and conſequently about 150000 in all: And I ſee no Reaſon why in all this and the other Plantations of *Aſia*, *Africa*, and *America*, there ſhould not be half a Million in all. But this laſt I leave to every Man's Conjecture, and conſequently I ſuppoſe that the King of *England* hath about 10 Millions of Subjects, *ubivis Terrarum Orbis*, and the King of *France* about 13$\frac{1}{2}$ as aforeſaid.

Although it be very material to know the Number of Subjects belonging to each Prince, yet when the Queſtion is concerning their Wealth and Strength, it is alſo material to examine, how many of them do get more than they ſpend, and how many leſs.

In Order whereunto it is be conſidered, that in the King of *England*'s Dominions, there are not 20000 Churchmen; but in *France*, as the afore-mentioned Author of theirs doth aver, (who ſets down the particular Number of each religious Order) there are about 270000; *viz.* 250000 more than we think are neceſſary, (that is to ſay) 250000 withdrawn out of the World. Now the ſaid Number of adult and able-bodied Perſons are equivalent to about double the ſame Number

Number of the promiſcuous Maſs of Mankind. And the ſame Author ſays, that the ſame religious Perſons do ſpend one with another about 18 *d. per Diem*, which is triple even to what a labouring Man requires.

Wherefore the ſaid 250000 Churchmen (living as they do) makes the King of *France*'s 13 Millions and a half, to be leſs than 13: Now if 10 Men can defend themſelves as well in Iſlands, as 13 can upon the Continent, then the ſaid 10 being not concerned to increaſe their Territory by the Invaſion of others, are as effectual as the 13 in Point of Strength alſo; wherefore that there are more Superlucrators in the *Engliſh* than the *French* Dominions, we ſay as followeth.

There be in *England*, *Scotland*, *Ireland*, and the King's other Territories, above 40000 Seamen; in *France* not above $\frac{1}{4}$ ſo many; but one Seaman earneth as much as 3 common Huſbandmen; wherefore this Difference in Seamen, added to the Account of the King of *England*'s Subjects, is an Advantage equivalent to 60000 Huſbandmen.

There are in *England*, *Scotland*, and *Ireland*, and all other the King of *England*'s Territories, 600000 Ton of Shipping, worth about 4$\frac{1}{2}$ Millions of Money; and the annual Charge of maintaining the Shipping of *England* by new Buildings and Reparations, is about $\frac{1}{4}$ Part of the ſame Sum; which is the Wages of 150000 Husbandmen, but is not the Wages of above $\frac{1}{3}$ Part of ſo many Artiſans as are employed up-

on Shipping of all Sorts; *viz.* Shipwrights, Calkers, Joyners, Carvers, Painters, Block-makers, Rope-makers, Maſt-makers, Smiths of ſeveral Sorts, Flag-makers, Compaſs-makers, Brewers, Bakers, and all other Sort of Victuallers; all Sorts of Tradeſmen relating to Guns, and Gunners Stores. Wherefore there being 4 Times more of theſe Artiſans in *England*, &c. than in *France*, they further add to the Account of the King of *England*'s Subjects, the Equivalent of 80000 Huſbandmen more.

The Sea-line of *England*, *Scotland*, and *Ireland*, and the adjacent Iſlands, is about 3800 Miles; according to which Length, and the whole Content of Acres, the ſaid Land would be an oblong or parallelogram Figure of 3800 Miles long, and about 24 Miles broad; and conſequently every Part of *England*, *Scotland*, and *Ireland*, is one with another, but 12 Miles from the Sea: Whereas *France* containing but about 1000 Miles of Sea-line, is by the like Method or Computation, about 65 Miles from the Sea-ſide; and conſidering the Paucity of Ports, in Compariſon of what are in the King of *England*'s Dominions, as good as 70 Miles diſtant from a Port: Upon which Grounds it is clear, that *England* can be ſupplied, with all groſs and bulky Commodities of foreign Growth and Manufacture, at far cheaper Rates than *France* can be, *viz.* at about 4 *s. per Cent.* cheaper; the Land-Carriage for the Difference of the Diſtance between *England* and *France*

from

from a Port, being ſo much or near thereabouts. Now to what Advantage this Conveniency amounteth, upon the Importation and Exportation of bulky Commodities, cannot be leſs than the Labour of one Million of People, *&c.* meaning by bulky Commodities all Sort of Timber, Plank, and Staves for Caſk; all Iron, Lead, Stones, Bricks, and Tyles for building; all Corn, Salt, and Drinks; all Fleſh and Fiſh, and indeed all other Commodities, wherein the Gain and Loſs of 4 *s. per Cent.* is conſiderable; where note that the like Wines are ſold in the inner Parts of *France* for 4 or 5 *l.* a Tun, which near the Ports yield 7 *l.* Moreover upon this Principle the Decay of Timber in *England* is no very formidable Thing, as the Rebuilding of *London*, and of the Ships waſted by the *Dutch* War do clearly manifeſt; nor can there be any Want of Corn, or other neceſſary Proviſions in *England*, unleſs the Weather hath been univerſally unſeaſonable for the Growth of the ſame; which ſeldom or never happens; for the ſame Cauſes which make Dearth in one Place, do often cauſe Plenty in another; wet Weather being propitious to Highlands, which drowneth the Low.

It is obſerved that the Poor of *France* have generally leſs Wages than in *England*, and yet their Victuals are generally dearer there; which being ſo, there may be more Superlucration in *England* than in *France*.

Laſtly, I offer it to the Conſideration of all thoſe, who have travelled through *England* and *France*; Whether the Plebeians of *England* (for they conſtitute the Bulk of any Nation) do not ſpend ⅓ Part more than the Plebeians of *France*? And if ſo, it is neceſſary that they muſt firſt get it; and conſequently that 10 Millions of the King of *England*'s Subjects are equivalent to 12 of the King of *France*; and upon the whole Matter, to the 13 Millions, at which the *French* Nation was eſtimated.

It will here be objected, that the Splendor and Magnificencies of the King of *France*, appearing greater than thoſe of *England*, that the Wealth of *France* muſt be proportionably greater than that of *England*; but that doth not follow, foraſmuch as the apparent Greatneſs of the King doth depend upon the *quota pars* of the People's Wealth which he levyeth from them; for ſuppoſing of the People to be equally rich, if one of the Sovereigns levy a fifth Part, and another a fifteenth, the one ſeems actually thrice as rich as the other, whereas potentially, they are but equal.

Having thus diſcourſed of the Territory, People, Superlucration, and Defenciblenes of both Dominions, and in ſome Meaſure of their Trade, ſo far as we had Occaſion to mention Ships, Shipping, and Nearneſs to Ports; we come next to enlarge a little further upon the Trade of each.

Some

Some have eſtimated, that there are not above 300 Millions of People in the whole World. Whether that be ſo or no, is not very material to be known; but I have fair Grounds to conjecture, and would be glad to know it more certainly, that there are not above 80 Millions with whom the *Engliſh* and *Dutch* have Commerce; no *Europeans* that I know of, trading directly or indirectly, where they do not; ſo as the whole commercial World, or World of Trade, conſiſteth of about 80 Millions of Souls, as aforeſaid.

And I further eſtimate, that the Value of all Commodities yearly exchanged amongſt them, doth not exceed the Value of 45 Millions: Now the Wealth of every Nation, conſiſting chiefly in the Share which they have in the Foreign Trade with the whole commercial World, rather than in the domeſtick Trade, of ordinary Meat, Drink, and Cloaths, *&c.* which bringing in little Gold, Silver, Jewels, and other univerſal Wealth; we are to conſider, whether the Subjects of the King of *England*, Head for Head, have not a greater Share than thoſe of *France*.

To which Purpoſe it hath been conſidered, that the Manufactures of Wool, yearly exported out of *England* into ſeveral Parts of the World, *viz.* All Sorts of Cloth, Serges, Stuffs, Cottons, Bayes, Sayes, Frize, Perpetuanas; as alſo Stockings, Caps, Rugs, *&c.* exported out of *England*, *Scotland*, and *Ireland*, do amount unto 5 Millions *per Annum*.

The Value of Lead, Tin, and Coals, to be 500,000 *l.*

The Value of all Cloaths, Houſhold-ſtuff, *&c.* carried into *America*, 200,000 *l.*

The Value of Silver and Gold taken from the *Spaniards* 60,000 *l.*

The Value of Sugar, Indigo, Tobacco, Cotton, and Cocoa, brought from the Southward Parts of *America* 600,000 *l.*

The Value of the Fiſh, Pipe-ſtaves, Maſts, Bever, *&c.* brought from *New England*, and the Northern Parts of *America* 200,000 *l.*

The Value of the Wool, Butter, Hides, Tallow, Beef, Herring, Pilchers, and Salmon, exported out of *Ireland*, 800,000 *l.*

The Value of the Coals, Salt, Linnen, Yarn, Herrings, Pilchers, Salmon, Linnen-Cloth, and Yarn, brought out of *Scotland* and *Ireland*, 500,000 *l.*

The Value of Salt-peter, Pepper, Callicoes, Diamonds, Drugs and Silks, brought out of the *Eaſt-Indies*, above what was ſpent in *England*, 800,000 *l.*

The Value of the Slaves, brought out of *Africa*, to ſerve in our *American* Plantations, 20000 *l*; which with the Freight of *Engliſh* Shipping, trading into foreign Parts, being above 1 Million and ½, makes in all 1918000 *l.*

Which Computation is ſufficiently juſtified by the Cuſtoms of the 3 Kingdoms, whoſe intrinſick Value are thought to be near a Million *per Annum*, *viz.* 600,000 *l.* payable to the King;

King; 100,000 *l.* for the Charges of Collecting, *&c.* 200,000 *l.* ſmuggled by the Merchants, and 100,000 *l.* gained by the Farmers; according to common Opinion and Mens Sayings: And this agrees alſo with that Proportion or Part of the whole Trade of the World, which I have eſtimated the Subjects of the King of *England* to be poſſeſſed of, *viz.* of about 10 of 45 Millions.

But the Value of the *French* Commodities brought into *England*, (notwithſtanding ſome current Eſtimates,) are not above 1200000 *l. per Annum*; and the Value of all they export into all the World beſides, not above 3 or 4 Times as much; which Computation alſo agreeth well enough with the Account we have of the Cuſtoms of *France*; ſo as *France* not exporting above $\frac{1}{3}$ the Value of what *England* doth, and for that all the Commodities of *France* (except Wines, Brandy, Paper, and the firſt Patterns and Faſhions for Cloaths and Furniture (of which *France* is the Mint) are imitable by the *Engliſh*, and having withal more People than *England*, it follows that the People of *England*, *&c.* have Head for Head, thrice as much Foreign Trade as the People of *France*; and about 2 Parts of 9 of the Trade of the whole commercial World; and about 2 Parts in 7 of all the Shipping: Notwithſtanding all which it is not to be denied, that the King and ſome great Men of *France*, appear more rich and ſplendid, than thoſe of the like Quality in *England*;

all which ariſes rather from the Nature of their Government, than from the intrinſick and natural Cauſes of Wealth and Power.

CHAP. V.

That the Impediments of England's *Greatneſs are but contingent and removable.*

THE firſt Impediment of *England*'s Greatneſs is, that the Territories thereunto belonging are too far aſunder, and divided by the Sea into many ſeveral Iſlands and Countries; and I may ſay, into ſo many Kingdoms, and ſeveral Governments, (*viz.*) there be three diſtinct Legiſlative Powers in *England*, *Scotland*, and *Ireland*; the which inſtead of uniting together, do often croſs one anothers Intereſt; putting Bars and Impediments upon one anothers Trades, not only as if they were Foreigners to each other, but ſometimes as Enemies.

2, The Iſlands of *Jerſey* and *Guernſey*, and the *Iſle of Man*, are under Juriſdictions different from thoſe either of *England*, *Scotland*, or *Ireland*.

3, The Government of *New England* (both civil and eccleſiaſtical) doth ſo differ from that of his Majeſty's other Dominions, that 'tis hard to ſay what may be the Conſequence of it.

And the Government of the other Plantations doth alſo differ very much from any of the reſt; although there be not naturally ſubſtantial Reaſons from the Situation, Trade, and Con-

Condition of the People, why there ſhould be ſuch Differences.

From all which it comes to paſs, that ſmall divided remote Governments being ſeldom able to defend themſelves, the Burthen of protecting of them all, muſt lye upon the chief Kingdom *England*; and ſo all the ſmaller Kingdoms and Dominions, inſtead of being Additions are really Diminutions; but the ſame is remedied by making two ſuch grand Councils, as may equally repreſent the whole Empire, one to be choſen by the King, the other by the People. The Wealth of a King is threefold, one is the Wealth of his Subjects, the ſecond is the *quota pars* of his Subjects Wealth, given him for the publick Defence, Honour, and Ornament of the People, and to manage ſuch Undertakings for the common Good, as no one or a few private Men are ſufficient for.

The third Sort are the *quota* of the laſt-mentioned *quota pars*, which the King may diſpoſe of as his own perſonal Inclination and Diſcretion ſhall direct him, without Account. Now it is moſt manifeſt, that the afore-mentioned Diſtances and Differences, of Kingdoms and Juriſdictions, are great Impediments to all the ſaid ſeveral Sorts of Wealth, as may be ſeen in the following Particulars. 1ſt, In Caſe of War with Foreign Nations, *England* commonly beareth the whole Burthen, and Charge, whereby many in *England* are utterly undone.

2dly,

2dly, *England* ſometimes prohibiting the Commodities of *Ireland* and *Scotland*, as of late it did the Cattle, Fleſh, and Fiſh, of *Ireland*, did not only make Food, and conſequently Labour dearer in *England*, but alſo hath forced the People of *Ireland* to fetch thoſe Commodities from *France*, *Holland*, and other Places, which before was ſold them from *England*, to the great Prejudice of both Nations.

3dly, It occaſions an unneceſſary Trouble and Charge, in collecting of Cuſtoms, upon Commodities paſſing between the ſeveral Nations.

4thly, It is a Damage to our *Barbadoes*, and other *American* Traders, that the Goods which might paſs thence immediately, to ſeveral Parts of the World, and to be ſold at moderate Rates, muſt firſt come into *England*, and there pay Duties, and afterwards (if at all) paſs into thoſe Countries, whither they might have gone immediately.

5thly, The Iſlands of *Jerſey* and *Guernſey*, are protected at the Charge of *England*, nevertheleſs the Labour and Induſtry of that People (which is very great) redounds moſt to the Profit of the *French*.

6thly, In *New England* there are vaſt Numbers of able-bodied *Engliſhmen*, employed chiefly in Huſbandry, and in the meaneſt Part of it, (which is breeding of Cattle) whereas *Ireland* would have contained all thoſe Perſons, and at worſt would have afforded them Lands on better Terms than they have them in *America*, if not

not ſome other better Trade withal, than now they can have.

7thly, The Inhabitants of the other Plantations, although they do indeed plant Commodities which will not grow ſo well in *England*; yet graſping at more Land, than will ſuffice to produce the ſaid Exotics in a ſufficient Quantity to ſerve the whole World, they do therein but diſtract and confound the Effect of their own Endeavours.

8thly, There is no Doubt that the ſame People, far and wide diſperſed, muſt ſpend more upon their Government and Protection, than the ſame living compactly, and when they have no Occaſion to depend upon the Wind, Weather, and all the Accidents of the Sea.

A ſecond Impediment to the Greatneſs of *England*, is the different underſtanding of ſeveral material Points, *viz.* Of the King's Prerogative, Privileges of Parliament, the obſcure Differences between Law and Equity; as alſo between civil and eccleſiaſtical Juriſdictions; Doubts whether the Kingdom of *England* hath Power over the Kingdom of *Ireland*, beſides the wonderful Paradox that *Engliſhmen*, lawfully ſent to ſuppreſs Rebellions in *Ireland*, ſhould after having effected the ſame, be (as it were) diſfranchiſed, and loſe that Intereſt in the Legiſlative Power which they had in *England*, and pay Cuſtoms as Foreigners for all they ſpend in *Ireland*, whither they were ſent for the Honour and Benefit of *England*.

The third Impediment is, That *Ireland* being a conquered Country, and containing not the Tenth Part as many *Iriſh* Natives as there are *Engliſh* in both Kingdoms, that natural and firm Union is not made, between the Two People, by Tranſplantations, and proportionable Mixture, ſo as there may be but a Tenth Part of the *Iriſh* in *Ireland*, and the ſame Proportion in *England*; whereby the Neceſſity of maintaining an Army in *Ireland*, at the Expence of a Quarter of the Rents of that Kingdom, may be taken away.

The fourth Impediment is, that Taxes in *England* are not levied upon the Expence, but upon the whole Eſtate; not upon Lands, Stock, and Labour, but chiefly upon Land alone; and that not by any equal and indifferent Standard, but the caſual Predominancy of Parties and Factions: And moreover, that theſe Taxes are not levied with the leaſt Trouble and Charge, but let out to Farmers, who alſo let them from one to another without explicit Knowledge of what they do; but ſo as in Concluſion, the poor People pay twice as much as the King receives.

The fifth Impediment is the Inequality of Shires, Dioceſſes, Pariſhes, Church-Livings, and other Precincts, as alſo the Repreſentation of the People in Parliament; all which do hinder the Operations of Authority in the ſame Manner as a Wheel irregularly made, and excentrically hung, neither moves ſo eaſily, nor performs its Work ſo truly, as if the ſame were duly framed and poiſed.

6thly,

6thly, Whether it be an Impediment, that the Power of making War, and raiſing Money, be not in the ſame Hand, much may be ſaid; but I leave it to thoſe who may more properly meddle with fundamental Laws.

None of theſe Impediments are natural, but did ariſe as the Irregularity of Buildings do, by being built Part at one Time, and Part at another; and by the changing of the State of Things from what they were at the reſpective Times, when the Practices we complain of were firſt admitted, and perhaps, are but the Warpings of Time, from the Rectitude of the firſt Inſtitution.

As theſe Impediments are contingent, ſo they are alſo removeable; for may not the Land of ſuperfluous Territories be ſold, and the People with their Moveables brought away? May not the *Engliſh* in the *American* Plantations (who plant Tobacco, Sugar, &c.) compute what Land will ſerve their Turn, and then contract their Habitations to that Proportion, both for Quantity and Quality? As for the People of *New England*, I can but wiſh they were tranſplanted into *Old England*, or *Ireland*, (according to Propoſals of their own, made within this 20 Years) although they were allowed more Liberty of Conſcience, than they allow one another.

May not the three Kingdoms be united into one, and equally repreſented in Parliament? Might not the ſeveral Species of the King's Subjects be equally mixed in their Habitations?

Might

Might not the Pariſhes and other Precincts, be better equalized? Might not Juriſdictions and Pretences of Power, be determined and aſcertained? Might not the Taxes be equally allotted, and directly applied to their ultimate Uſe? Might not Diſſenters in Religion be indulged, they paying for a competent Force to keep the publick Peace? I humbly venture to ſay all theſe Things may be done, if it be ſo thought fit by the Sovereign Power, becauſe the like hath often been done already, at ſeveral Places and Times.

CHAP. VI.

That the Power and Wealth of England, *hath increaſed this laſt forty Years.*

IT is not much to be doubted, but that the Territories under the King's Dominions have increaſed; foraſmuch as *New England*, *Virginia*, *Barbadoes*, and *Jamaica*, *Tangier*, and *Bombay*, have ſince that Time, been either added to his Majeſty's Territories, or improved from a deſart Condition, to abound with People, Buildings, Shipping, and the Production of many uſeful Commodities. And as for the Land of *England*, *Scotland*, and *Ireland*, as it is not leſs in Quantity than it was forty Years ſince, ſo it is manifeſt that by Reaſon of the draining of Fens, watering of dry Grounds, improving of Forreſts and Commons, making of heathy and barren Grounds to bear Saintfoyne and Clover-graſs; meliora-

meliorating and multiplying ſeveral Sorts of Fruits, and Garden-ſtuff, making ſome Rivers navigable, &c. I ſay it is manifeſt, that the Land in its preſent Condition, is able to bear more Proviſion and Commodities, than it was forty Years ago.

2dly, Altho' the People in *England*, *Scotland*, and *Ireland*, which have extraordinarily periſhed by the Plague and Sword, within this laſt forty Years, do amount to about 300000, above what have died in the ordinary Way; yet the ordinary Increaſe by Generation of 10 Millions, which doubles in two hundred Years, as hath been ſhewn by the Obſervators upon the Bills of Mortality, may in forty Years (which is a fifth Part of the ſame Time) have increaſed one fifth Part of the whole Number, or two Millions. Where note by the Way, that the Acceſſion of Negroes to the *American* Plantations (being all Men of great Labour and little Expence) is not inconſiderable; beſides it is hoped that *New England*, where few or no Women are barren, and moſt have many Children, and where People live long, and healthfully, hath produced an Increaſe of as many People as were deſtroyed in the late Tumults in *Ireland*.

As for Houſing, the Streets of *London* itſelf ſpeaks it, I conceive it is double in Value in that City, to what it was forty Years ſince; and for Houſing in the Country, they have increaſed at *Newcaſtle*, *Yarmouth*, *Norwich*, *Exeter*, *Portſmouth*, *Cowes*, *Dublin*, *Kinſale*, *Londonderry*, and *Cole-*

Coleraine in *Ireland*, far beyond the Proportion of what I can learn have been dilapidated in other Places. For in *Ireland* where the Ruin was greateſt, the Houſing (taking all together) is now more valuable than forty Years ago, nor is this to be doubted, ſince Houſing is now more ſplendid than in thoſe Days, and the Number of Dwellers is increaſed by near one fifth Part; as in the laſt Paragraph is ſet forth.

As for Shipping, his Majeſty's Navy is now triple, or quadruple, to what it was forty Years ſince, and before the *Sovereign* was built; the Shipping trading to *Newcaſtle*, which are now about eighty thouſand Tons, could not be then above a Quarter of that Quantity. Firſt, becauſe the City of *London* is doubled. 2. Becauſe the Uſe of Coals is alſo at leaſt doubled, becauſe they were heretofore ſeldom uſed in Chambers, as now they are, nor were there ſo many Bricks burned with them as of late, nor did the Country on both Sides the *Thames* make uſe of them as now. Beſides there are employed in the *Guiney* and *American* Trade above Forty thouſand Ton of Shipping *per Annum*; which Trade in thoſe Days was inconſiderable. The Quantity of Wines imported, was not near ſo much as now; and to be ſhort, the Cuſtoms upon imported and exported Commodities, did not then yield a third Part of the preſent Value; which ſhews that not only Shipping, but Trade itſelf hath increaſed ſomewhat near that Proportion.

As

As to Money, the Intereſt thereof was within this fifty Years at 10 *l. per Cent.* forty Years ago at 8 *l.* and now at 6 *l.* no Thanks to any Laws which have been made to that Purpoſe, foraſmuch as thoſe who can give good Security, may now have it at leſs: But the natural Fall of Intereſt, is the Effect of the Increaſe of Money.

Moreover if rented Lands and Houſes have increaſed; and if Trade hath increaſed alſo, it is certain that Money which payeth thoſe Rents, and driveth on Trade, muſt have increaſed alſo.

Laſtly, I leave it to the Conſideration of all Obſervers, whether the Number and Splendor of Coaches, Equipage, and Houſhold Furniture, hath not increaſed ſince that Time; to ſay nothing of the Poſtage of Letters, which have increaſed from one to twenty, which argues the Increaſe of Buſineſs and Negotiation. I might add that his Majeſty's Revenue is near tripled, and therefore the Means to pay, and bear the ſame, have increaſed alſo.

CHAP. VII.

That one tenth Part of the whole Expence of the King of England's *Subjects, is ſufficient to maintain one hundred Thouſand Foot, forty Thouſand Horſe, and forty Thouſand Men at Sea; and defray all other Charges of the Government, both ordinary and extraordinary, if the ſame were regularly taxed and raiſed.*

TO clear this Point, we are to find out what is the middle Expence of each Head in the King's Dominions, between the higheſt and the loweſt; to which I ſay it is not probably leſs than the Expence of a Labourer, who earneth about 8 *d. per* Day; for the Wages of ſuch a Man is 4 *s. per* Week without Victuals, or 2 *s.* with it; wherefore the Value of his Victuals is 2 *s. per* Week, or 5 *l.* 4 *s. per Annum:* Now the Value of Cloaths cannot be leſs than the Wages given to the pooreſt Maid-Servant in the Country, which is 30 *s. per Annum*, nor can the Charge of all other Neceſſaries be leſs than 6 *l. per Annum* more; wherefore the whole Charge is 7 *l.* 10 *s.*

It is not likely that this Diſcourſe will fall into the Hands of any that live at 7 *l. per Annum*, and therefore ſuch will wonder at this Suppoſition: But if they conſider how much the Number of the Poor, and their Children, is greater than that of the Rich; although the perſonal Expence of ſome rich Men ſhould be

twenty

twenty Times more than that of a Labourer; yet the Expence of the Labourer above-mentioned may well enough ſtand for the Standard of the Expence of the whole Maſs of Mankind.

Now if the Expence of each Man, one with another, be 7 *l. per Annum*, and if the Number of the King's Subjects be ten Millions, then the tenth Part of the whole Expence will be ſeven Millions; but about five Millions, or a very little more, will amount to one Year's Pay for one hundred thouſand Foot, forty thouſand Horſe, and forty thouſand Men at Sea, Winter and Summer; which can rarely be neceſſary. And the ordinary Charge of the Government, in Times of deep and ſerene Peace, was not ſix hundred thouſand Pound *per Annum*.

Where a People thrive, there the Income is greater than the Expence, and conſequently the tenth Part of the Expence is not a tenth Part of the Income; now for Men to pay a Tenth of their Expence, in a Time of the greateſt Exigency, (for ſuch it muſt be when ſo great Forces are requiſite) can be no Hardſhip, much leſs a deplorable Condition; for to bear the tenth Part, a Man needs ſpend but a twentieth Part leſs, and labour a twentieth Part more, or half an Hour *per Diem* extraordinary, both which, within common Experience, are very tolerable; there being very few in *England* who do not eat by a twentieth Part more than does them good; and what Miſery were it, inſtead of

wearing Cloth of 20 *s. per* Yard, to be contented with that of 19 *s.* few Men having Skill enough to diſcern the Difference.

Memorandum, That all this while I ſuppoſe that all of theſe ten Millions of People are obedient to their Sovereign, and within the Reach of his Power; for as Things are otherwiſe, ſo the Calculation muſt be varied.

CHAP. VIII.

That there are ſpare Hands enough among the King of England's *Subjects, to earn two Millions* per Annum *more than they now do; and that there are alſo Employments ready, proper, and ſufficient, for that Purpoſe.*

TO prove this Point we muſt enquire, how much all the People could earn, if they were diſpoſed or neceſſitated to labour, and had Work whereupon to employ themſelves; and compare that Sum with that of the total Expence above mentioned, deducting the Rents and Profits of their Land and Stock, which, properly ſpeaking, ſaveth ſo much Labour. Now the Proceed of the ſaid Lands and Stock in the Countries, is about three Parts of ſeven of the whole Expence; ſo as where the Expence is ſeventy Millions, the Rent of the Land, and the Profit of all the perſonal Eſtate, Intereſt of Money, *&c.* muſt be about thirty Millions; and conſequently, the Value of the Labour forty Millions, that is 4 *l. per* Head.

But

But it is to be noted, That about a Quarter of the Maſs of Mankind are Children, Males and Females, under ſeven Years old, from whom little Labour is to be expected. It is alſo to be noted, that about another tenth Part of the whole People are ſuch as, by Reaſon of their great Eſtates, Titles, Dignities, Offices, and Profeſſions, are exempt from that Kind of Labour we now ſpeak of; their Buſineſs being, or ought to be, to govern, regulate, and direct the Labours and Actions of others. So that of ten Millions, there may be about ſix Millions and an half, which (if Need require) might actually labour: And of theſe ſome might earn 3 *s. per* Week, ſome 5 *s.* and ſome 7 *s.* That is, all of them might earn 5 *s. per* Week at a Medium one with another; or at leaſt 10 *l. per Annum*, (allowing for Sickneſs, and other Accidents;) whereby the whole might earn ſixty five Millions *per Annum*, that is twenty five more than the Expence.

The Author of the State of *England* ſays that the Children of *Norwich*, between ſix and ſixteen Years old, do earn 12000 *l. per Annum*, more than they ſpend. Now foraſmuch as the People of *Norwich* are a three hundredth Part of all the People of *England*, as appears by the Accompts of the Hearth-money, and about a five hundredth Part of all the King's Subjects throughout the World; it follows that all his Majeſty's Subjects between ſix and ſixteen Years

old, may earn five Millions *per Annum* more than they ſpend.

Again, foraſmuch as the Number of People above ſixteen Years old, are double the Number of thoſe between ſix and ſixteen, and that each of the Men can earn double to each of the Children; it is plain that if the Men and Children every where did work as they do in *Norwich*, they might earn twenty five Millions *per Annum* more than they ſpend; which Eſtimate, grounded upon Matter of Fact and Experience, agrees with the former.

Although as hath been proved, the People of *England* do thrive, and that it is poſſible they might ſuperlucrate twenty five Millions *per Annum*; yet it is manifeſt that they do not, nor twenty three, which is leſs by the two Millions herein meant; for if they did ſuperlucrate twenty three Millions, then in about five or ſix Years Time, the whole Stock and perſonal Eſtate of the Nation would be doubled, which I wiſh were true, but find no Manner of Reaſon to believe; wherefore if they can ſuperlucrate twenty five, but do not actuall ſuperlucrate twenty three, nor twenty, nor ten, nor perhaps five, I have then proved what was propounded; *viz.* That there are ſpare Hands among the King's Subjects to earn, at leaſt, two Millions more than they do.

But to ſpeak a little more particularly concerning this Matter, It is to be noted, that

ſince

ſince the Fire of *London*. there was earned in four Years by Tradeſmen (relating to Building only) the Sum of four Millions; *viz.* one Million *per Anuum*, without leſſening any other Sort of Work, Labour, or Manufacture, which was uſually done in any other four Years before the ſaid Occaſion. But if the Tradeſmen relating to Building only, and ſuch of them only as wrought in and about *London*, could do one Million worth of Work extraordinary; I think that from thence, and from what hath been ſaid before, all the reſt of the ſpare Hands might very well double the ſame, which is as much as was propounded.

Now if there were ſpare Hands to ſuperlucrate Millions upon Millions, they ſignify nothing unleſs there were Employment for them; and may as well follow their Pleaſures and Speculations as labour to no Purpoſe; therefore the more material Point is, to prove that there is two Millions Worth of Work to be done, which at preſent the King's Subjects do neglect.

For the Proof of this there need little more to be done, than to compute 1. How much Money is paid by the King of *England*'s Subjects to Foreigners for Freight of Shipping. 2. How much the *Hollanders* gain by their Fiſhing Trade practiſed upon our Seas. 3. What the Value is of all the Commodities imported into and ſpent in *England*, which might by Diligence be produced and manufactured here. To

make ſhort of this Matter, upon Peruſal of the moſt authentick Accompts relating to theſe ſeveral Particulars, I affirm that the ſame amounteth to above five Millions, whereas I propounded but two Millions.

For a further Proof whereof Mr. *Samuel Fortry* in his ingenious Diſcourſe of Trade, exhibits the Particulars, wherein it appears, that the Goods imported out of *France* only, amount yearly to two Millions ſix hundred thouſand Pounds. And I affirm, that the Wine, Paper, Cork, Roſin, Capers, and a few other Commodities which *England* cannot produce, do not amount to one fifth Part of the ſaid Sum. From whence it follows, that (if Mr. *Fortry* hath not erred) the two Millions here mentioned may ariſe from *France* alone; and conſequently five or ſix Millions from all the three Heads laſt above ſpecified.

CHAP. IX.

That there is Money ſufficient to drive the Trade of the Nation.

SINCE his Majeſty's happy Reſtauration, it was thought fit to call in, and new coin, the Money which was made in the Times of Uſurpation. Now it was obſerved by the general Conſent of Caſhiers, that the ſaid Money (being by frequent Revolutions well mixed with old)

old) was about a ſeventh Part thereof; and that the ſaid Money being called in, was about Eight hundred thouſand Pounds, and conſequently the Whole five Millions ſix hundred thouſand Pound. Whereby it is probable that (ſome Allowance being given for hoarded Money) the whole Caſh of *England* was then about ſix Millions, which I conceive is ſufficient to drive the Trade of *England*, not doubting but the reſt of his Majeſty's Dominions have the like Means to do the ſame reſpectively.

If there be ſix Millions of Souls in *England*, and that each ſpendeth 7 *l. per Annum*, then the whole Expence is forty two Millions, or about Eight hundred thouſand Pound *per* Week; and conſequently, if every Man did pay his Expence weekly, and that the Money could circulate within the Compaſs of a Week, then leſs than one Million would anſwer the Ends propoſed. But foraſmuch as the Rents of the Lands in *England* (which are paid half-yearly) are eight Millions *per Annum*, there muſt be four Millions to pay them. And foraſmuch as the Rent of the Houſing of *England*, paid quarterly, are worth about four Millions *per Annum*, there needs but one Million to pay the ſaid Rents; wherefore ſix Millions being enough to make good the three Sorts of Circulations above mentioned, I conceive what was propoſed is competently proved, at leaſt

until

until ſomething better be advanced to the contrary.

CHAP. X.

That the King of England's *Subjects have Stock competent and convenient to drive the Trade of the whole commercial World.*

NOW for the further Encouragement of Trade, as we have ſhewn that there is Money enough in *England* to manage the Affairs thereof; ſo we ſhall now offer to Conſideration, whether there be not a competent and convenient Stock to drive the Trade of the whole commercial World. To which Purpoſe it is to be remember'd, that all the Commodities yearly exported out of every Part of the laſt mentioned World, may be bought for forty five Millions; and that the Shipping employed in the ſame World are not worth above fifteen Millions more, and conſequently, that ſixty Millions at moſt, would drive the whole Trade above mentioned, without any Truſt at all. But foraſmuch as the Growers of Commodities do commonly truſt them to ſuch Merchants or Factors as are worth but ſuch a Part of the full Value of their Commodities as may poſſibly be loſt upon the Sale of them, whereas Gain is rather to be expected; it follows that leſs than a Stock of Sixty Millions, nay leſs than Half of the ſame Sum, is ſufficient to drive the Trade above

above mentioned: It being well known that any Tradeſman of good Reputation, worth Five hundred Pounds, will be truſted with above One thouſand Pounds Worth of Commodities: Wherefore leſs than thirty Millions will ſuffice for the ſaid Purpoſe; of which Sum the Coin, Shipping, and Stock, already in Trade, do at leaſt make one Half.

And it hath been ſhewn how by the Policy of a Bank, any Sum of Money may be made equivalent in Trade, unto near double of the ſame; by all which it ſeems, that even at preſent much is not wanting to perform what is propounded. But ſuppoſe twenty Millions or more were wanting, it is not improbable, that ſince the Generality of Gentlemen, and ſome Noblemen, do put their younger Sons to Merchandize, they will ſee it reaſonable, as they increaſe in the Number of Merchants, ſo to increaſe the Magnitude of Trade, and conſequently to increaſe Stock; which may effectually be done by imbanking twenty Millions Worth of Land, not being above a Sixth or Seventh of the whole Territory of *England*; (that is to ſay) by making a Fund of ſuch Value, to be Security for all Commodities bought and ſold upon the Accompt of the univerſal Trade here mentioned.

And thus it having appeared, that *England* having in it as much Land, like *Holland* and *Zealand*, as the ſaid two Provinces do themſelves con-

contain, with Abundance of other Land not inconvenient for Trade, and that there are ſpare Hands enough to earn many Millions of Money more than they now do, and that there is alſo Employment to earn ſeveral Millions (even from the Conſumption of *England* itſelf) it follows from thence, and from what hath been ſaid in the laſt Paragraph, about enlarging of Stock both of Money and Land, that it is not an impoſſible, nay a very feaſible Matter, for the King of *England*'s Subjects to gain the univerſal Trade of the whole commercial World.

Nor is it unſeaſonable to intimate this Matter, foraſmuch as the younger Brothers of the good Families of *England* cannot otherwiſe be provided for, ſo as to live according to their Birth and Breeding: For if the Lands of *England* are worth eight Millions *per Annum*, then there be at a Medium about ten thouſand Families of about 800 *l. per Annum*; in each of which, one with another, we may ſuppoſe there is a younger Brother, whom leſs than two or three hundred Pounds *per Annum* will not maintain ſuitable to his Relations: Now I ſay that neither the Offices at Court, nor Commands in our ordinary Army and Navy, nor Church Preferments, nor the uſual Gains by the Profeſſion of the Law and Phyſick, nor the Employments under Noblemen and Prelates, will, all of them put together, furniſh

niſh Livelyhoods of above 300 *l. per Annum*, to three Thouſand of the ſaid ten Thouſand younger Brothers: Wherefore it remains that Trade alone muſt ſupply the reſt. But if the ſaid ſeven thouſand Gentlemen be applied to Trade, without increaſing of Trade, or if we hope to increaſe Trade without increaſing of Stock, which for ought appears is only to be done by imbanking a due Proportion of Lands, and Money, we muſt neceſſarily be diſappointed. Where note, that ſelling of Lands to Foreigners for Gold and Silver, would inlarge the Stock of the Kingdom. Whereas doing the ſame between one another, doth effect nothing: For he that turneth all his Land into Money, diſpoſes himſelf for Trade; and he that parteth with his Money for Land, doth the contrary: But to ſell Land to Foreigners, increaſeth both Money and People, and conſequently Trade. Wherefore it is to be thought, that when the Laws denying Strangers to purchaſe, and not permitting them to trade, without paying extraordinary Duties, were made; that then the publick State of Things, and Intereſt of the Nation were far different from what they now are.

Having handled theſe ten principal Concluſions, I might go on with others *ad infinitum*; but what hath been already ſaid I look upon as ſufficient to ſhew what I mean by *Political Arith-*

Arithmetick; and to ſhew the Uſes of knowing the true State of the People, Land, Stock, Trade, *&c.* 2. That the King's Subjects are not in ſo bad a Condition as diſcontented Men would make them. 3. To ſhew the great Effect of Unity, Induſtry, and Obedience, in Order to the common Safety, and each Man's particular Happineſs.

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I can have no Pretence to ſeek, elſewhere, for a Patron to this Third Edition, which the Demand I have met with for the Copies, and ſome typographical Errors, heretofore overlooked, have rendred neceſſary; and therefore I again treſpaſs on your Lord-

Lordſhip's Indulgence in this Addreſs, well knowing, that your uſual Candour and Goodneſs will excuſe any Imperfections that may ſtill remain in the Performance of

MY LORD,

Your Lordſhip's

moſt Obedient, and

moſt humble Servant,

A. DE MOIVRE.

PREFACE.

DR. *Halley* publiſhed in the *Philoſophical Tranſactions*, N°. 196. an Eſſay concerning the Valuation of Lives; it was partly built upon five Years Obſervation of the Bills of Mortality taken at *Breſlaw*, the Capital of *Sileſia*, and partly on his own Calculation.

Altho' he had thereby confirm'd the great Opinion which the World entertain'd of his Skill and Sagacity, yet he was ſenſible, that his Tables and Calculations were ſuſceptible of farther Improvements; of this he expreſs'd his Senſe, in the following Words; *Were this Calculus founded on the Experience of a very great Number of Years, it would very well be worth the while, to think of Methods to facilitate the Computation of two, three, or more Lives.*

From

From whence it appears, that the Table of Obſervations being only the Reſult of a few Years Experience, it was not ſo entirely to be depended upon, as to make it the Foundation of a fix'd and unalterable Valuation of Annuities on Lives; and that even admiting ſuch a Table could be obtain'd, as might be grounded on the Experience of a great Number of Years, ſtill the Method of applying it to the Valuation of ſeveral Lives, would be extremely laborious, conſidering the vaſt Number of Operations, that would be requiſite to combine every Year of each Life with every Year of all the other Lives.

The Subject of Annuities on Lives, had been long neglected by me, partly prevented by other Studies, partly wanting the neceſſary means to treat of it as it deſerved: But two or three Years after the Publication of the firſt Edition of my *Doctrine of Chances*, I took the Subject into conſideration; and conſulting Dr. *Halley*'s Table of Obſervations, I found that the Decrements of Life, for conſiderable Intervals of Time, were in Arithmetic Progreſſion; for Inſtance, out of 646 Perſons of twelve Years of Age, there remain 640 after one Year; 634 after two Years; 628, 622, 616, 610, 604, 598, 592, 586, after 3, 4, 5, 6, 7, 8, 9, 10 Years reſpectively, the common Difference of thoſe Numbers being 6.

Exa-

Examining afterwards other Caſes, I found that the Decrements of Life for ſeveral Years were ſtill in Arithmetic Progreſſion; which, may be obſerved from the Age of 54, to the Age of 71, where the Difference for 17 Years together, is conſtantly 10.

After having thoroughly examined the Tables of Obſervation, and diſcovered that Property of the Decrements of Life, I was inclined to compoſe a Table of the Values of Annuities on Lives, by keeping cloſe to the Tables of Obſervation; which would have been done with Eaſe, by taking, in the whole Extent of Life, ſeveral Intervals whether equal or unequal: However, before I undertook the Taſk, I tried what would be the Reſult, of ſuppoſing thoſe Decrements uniform from the Age of Twelve; being ſatisfied that the Exceſſes ariſing on one ſide, would be nearly compenſated by the Defects on the other; then comparing my Calculation with that of Dr. *Halley*, I found the Concluſion ſo very little different, that I thought it ſuperfluous to join together ſeveral different Rules, in order to compoſe a ſingle one: I need not take Notice, that from the Time of Birth to the Age of Twelve, the Probabilities of Life increaſe, rather than decreaſe, which is a Reaſon of the apparent Irregularity of the Tables in the beginning.

Another thing was neceſſary to my Calculation, which was, to ſuppoſe the Extent of Life confin'd to a certain Period of Time, which I ſuppoſed to be at 86: What induced me to aſſume that Suppoſition was, 1ſt, That Dr. *Halley* terminates his Tables of Obſervations at the 84th Year; for altho' out of 1000 Children of one Year of Age, there are twenty, who, according to Dr. *Halley*'s Tables, attain to the Age of 84 Years, which Number of 20, is inconſiderable, and would ſtill have been reduced, if the Obſervations had been carried two Years farther. 2d. It appears from the Tables of *Graunt*, who printed the firſt Edition of his Book above 80 Years ago, that out of 100 new-born Children, there remain'd not one after 86 Years; this was deduced from the Obſervations of ſeveral Years, both in the City and the Country, at a Time when the City being leſs populous, there was a greater Facility of coming at the Truth, than at preſent. 3d. I was farther confirm'd in my Hypotheſis, by Tables of Obſervation made in *Switzerland*, about the Beginning of this Century, wherein the Limit of Life is placed at 86; As for what is alledged, that by ſome Obſervations of late Years, it appears, that Life is carried to 90, 95, and even to 100 Years; I am no more moved by it, than by the Examples of *Parr*, or *Jenkins*, the firſt of which lived 152 Years, and the other 167.

To

To this may be added, that the Age of purchaſing Annuities for Life, ſeldom exceeds 70, at which Term, Dr. *Halley* ends his Tables of the Valuation of Lives.

The greateſt Difficulty that occurred to me in this Speculation, was to invent practical Rules that might eaſily be applied to the Valuation of ſeveral Lives; which, however, was happily overcome, the Rules being ſo eaſy, that by the Help of them, more can be perform'd in a Quarter of an Hour, than by any Method before extant, in a Quarter of a Year.

Since the Publication of my Firſt Edition, which was in 1724, I made ſome Improvements to it, as may be ſeem in the Second Edition of my *Doctrine of Chances*; but this Edition of Annuities has many Advantages over the former, and that in reſpect to the Diſpoſition of the Precepts, the Conciſeneſs of the Rules, the Multiplicity of Problems, and Uſefulneſs of the Tables I have invented.

Before I make an End of this Preface, I think it proper to obſerve, that altho' I have given Rules for finding the Value of Annuities for any Rate of Intereſt, yet I have confined myſelf in my Tables, to the ſeveral Rates of 4, 5 and 6 *per Cent.* which may be interpreted, as if I thought reaſonable, that when Land

ſcarce produces three and a half *per Cent.* and South-Sea Annuities barely that Intereſt, yet the Purchaſer of an Annuity ſhould make 4 *per Cent.* or above; but thoſe Caſes can hardly admit of Compariſon, it being well known, that Land in Fee-ſimple, procures to the Proprietor Credit, Honour, Reputation, and other Advantages, in conſideration of which, he is contented with a ſmaller Income. As to the Value of South-Sea Annuities, it has its Foundation on the Punctuality of Payments, and on a Parliamentary Security; but Annuities on Lives, have not the former Security, and ſeldom the latter.

ERRATA.

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THE

THE VALUATION OF ANNUITIES ON LIVES.

BEFORE I come to the Solution of Queſtions on Lives, it will be neceſſary to explain the Meaning of ſome Words which I ſhall often have occaſion to mention.

1°. SUPPOSING the *Probabilities of Life* to decreaſe in Arithmetic Progreſſion in ſuch manner, as that ſuppoſing, for Inſtance, 36 Perſons each of

the Age of 50, if after one Year expired there remains but 35, after two 34, after three 33, and ſo on; it is very plain that ſuch Lives would neceſſarily be extinct in 36 Years, and that therefore the *Probabilities of living* 1, 2, 3, 4, 5, *&c.* Years from this Age of 50 would fitly be repreſented by the Fractions $\frac{35}{36}$, $\frac{34}{36}$, $\frac{33}{36}$, $\frac{32}{36}$, $\frac{31}{36}$, *&c.* which decreaſe in Arithmetic Progreſſion.

I will not ſay that the Decrements of Life are preciſely in that Proportion, ſtill comparing that Hypotheſis with the Table of Dr. *Halley*, from the Obſervations made at *Breſlaw*, they will be found to be exceedingly approaching.

2°. I call that the *Complement of Life*, which remains from the Age given, to the Time of the Extinction of Life, which will be at 86, according to our Hypotheſis. Thus ſuppoſing an Age of 50, becauſe the Difference between 50 and 86 is 36, I call 36 the *Complement of Life.*

3°.

3°. I call that the *Rate of Intereſt*, which is properly the Amount of one Pound, put out at Intereſt for one Year, otherwiſe one Pound joined with the Intereſt it produces in one Year; thus ſuppoſing Intereſt at 5 *per Cent.* the Intereſt of 1 *l.* would be 0.05, which being joined to the Principal 1, produces 1.05, which is what I call the *Rate of Intereſt*.

PROBLEM I.

Suppoſing the Probabilities of Life to decreaſe in Arithmetic Progreſſion, to find the Value of an Annuity upon a Life of an Age given.

SOLUTION.

LET the Rent or Annuity be ſuppoſed $= 1$, the Rate of Intereſt $= r$, the Complement of Life $= n$, the Value of an Annuity certain to continue during n Years $= P$, then will the Value of the Life be $\frac{1 - \frac{r}{n} P}{r - 1}$, which is thus expreſſed in Words at length;

Take the Value of an Annuity certain for ſo many Years, as are denoted

by the Complement of Life; multiply this Value by the Rate of Intereſt, and divide the Product by the Complement of Life, then let the Quotient be ſubtracted from Unity, and let the Remainder be divided by the Intereſt of 1 l. *then this laſt Quotient will expreſs the Value of an Annuity for the Age given.*

THUS ſuppoſe it were required to find the preſent Value of an Annuity of 1 *l.* for an Age of 50, Intereſt being at 5 *per Cent.*

THE Complement of Life being 36, let the Value of an Annuity certain, according to the given Rate of Intereſt, be taken out of the Tables annexed to this Book, this Value will be found to be 16.5468.

LET this Value be multiplied by the Rate of Intereſt 1.05, the Product will be 17.3741.

LET this Product be divided by the Complement of Life, *viz.* by 36, the Quotient will be 0.04826.

SUB-

SUBTRACT this Quotient from Unity, the Remainder will be 0.5174.

LASTLY, divide this Quotient by the Intereſt of 1 *l. viz.* by 0.05, and the new Quotient will be 10.35; which will expreſs the Value of an Annuity of 1 *l.* or how many Years Purchaſe the ſaid Life of 50 is worth.

AND in the ſame manner, if Intereſt of Money was at 6 *per Cent.* an Annuity upon an Age of 50, would be found worth 9.49 Years Purchaſe.

BUT as I have annexed to this Treatiſe the Values of Annuities for an Intereſt of 4, 5, and 6 *per Cent.* it will not be neceſſary to calculate thoſe Caſes, but ſuch only as require a Rate of Intereſt higher or lower, or intermediate; which will ſeldom happen, but in caſe it does, the Rule may eaſily be applied.

PROBLEM II.

The Values of two ſingle Lives being given, to find the Value of an Annuity granted for the Time of their joint continuance.

SO-

SOLUTION.

LET M be the Value of one Life, P the Value of the other, r the Rate of Intereſt; then the Value of an Annuity upon the two joint Lives will be $\frac{MP}{\overline{M+P}-r-1MP}$, in Words thus;

Multiply together the Values of the two Lives, and reſerve the Product.

Let that Product be again multiplied by the Intereſt of 1 l. *and let that new Product be ſubtracted from the Sum of the Values of the Lives, and reſerve the Remainder.*

Divide the firſt Quantity reſerved by the ſecond, and the Quotient will expreſs the Value of the two joint Lives.

THUS, ſuppoſing one Life of 40 Years of Age, the other of 50, and Intereſt at 5 *per Cent.* The Value of the firſt Life will be found in the Tables to be 11.83, the Value of the ſecond 10.35, the Product will be 122.4405, which Product muſt be reſerved.

MUL-

MULTIPLY this again by the Intereſt of 1 *l. viz.* by 0.05, and this new Product will be 6.122025.

THIS new Product being ſubtracted from the Sum of the Lives which is 22.18, the Remainder will be 16.057975, and this is the ſecond Quantity reſerved.

NOW dividing the firſt Quantity reſerved by the ſecond, the Quotient will be 7.62 nearly; and this expreſſes the Values of the two joint Lives.

IF the Lives are equal, the Canon for the Value of the joint Lives will be ſhortened and be reduced to $\frac{M}{2-r-1\times M}$, which in words may be thus expreſſed;

Take the Value of one Life, and reſerve that Value.

Multiply this Value by the Intereſt of 1 l. *and then ſubtract the Product from the Number* 2, *and reſerve the Remainder.*

Divide the firſt Quantity reſerved by the ſecond, and the Quotient will expreſs the Value of the two equal joint Lives.

THUS,

THUS, ſuppoſing each Life to be 45 Years of Age, and Intereſt at 5 *per Cent.*

THE Value of one Life will be found to be 11.14, the firſt Quantity reſerved.

THIS being multiplied by 0.05 the Intereſt of 1 *l.* the Product will be 0.557.

THIS Product being ſubtracted from the Number 2, the Remainder will be 1.443, the ſecond Quantity reſerved.

DIVIDE the firſt Quantity reſerved *viz.* 11.14; by the ſecond, *viz.* 1.443, and the Quotient 7.72 will be the Value of the two joint Lives, each of 45 Years of Age.

PROBLEM III.

The Values of three ſingle Lives being given, to find the Value of an Annuity for the Time of their joint continuance.

SOLUTION.

LET *M*, *P*, *Q*, be the reſpective Values of the ſingle Lives, then the Values

Value of the three joint Lives will be $\frac{MPQ}{MP+MQ+PQ-2dMPQ}$, ſuppoſing *d* to repreſent the Intereſt of 1 *l.* in Words thus;

Multiply the Values of the ſingle Lives together, and reſerve the Product.

Let that Product be multiplied again by the Intereſt of 1 l. *and let the Double of that new Product be ſubtracted from the Sum of the ſeveral Products of the Lives taken two and two, and reſerve the Remainder.*

Divide the firſt Quantity reſerved by the ſecond, and the Quotient will be the Value of the three joint Lives.

THUS, ſuppoſing one Life to be worth 13 Years Purchaſe, the ſecond 14, the third 15, and Intereſt at 4 *per Cent*; the Product of the three Lives will be 2730, which being multiplied by the Intereſt of 1 *l. viz.* by 0.04, the new Product will be 109.20,

whereof the double is 218.40: Now the Product of the firſt Life by the ſecond is 182; the Product of the firſt Life by the third is 195; and the Product of the ſecond Life by the third is 210, the Sum of all which is 587; from which ſubtracting the Number 218.40 found above, the Remainder will be 368.60, by which the Product of the three Lives, *viz.* 2730 being divided, the Quotient 7.41 will be the Value of the three joint Lives.

BUT if the three Lives were equal, the general Expreſſion of the Value of the joint Lives will be much ſhorter, for let M repreſent the Value of one Life, d the Intereſt of 1 *l.* then the Value of the three joint Lives will be $\frac{M}{3-2dM}$, in Words thus;

Take the Value of one Life, and reſerve it, multiply this Value by the Intereſt of 1 l. *and double the Product.*

Sub-

Subtract this double Product from the Number 3, *and reſerve the Remainder.*

Divide the firſt Quantity reſerved by the ſecond, and the Quotient will be the Value of the three joint Lives.

THUS, ſuppoſing three equal Lives each worth 14 Years Purchaſe, reſerve the Number 14.

MULTIPLY this by 0.04, Intereſt of 1 *l.* the Product will be 0.56, which being doubled, will be 1.12.

THIS being ſubtracted from the Number 3, the Remainder will be 1.88 which is the ſecond Quantity to be reſerved.

DIVIDE 14 firſt Quantity reſerved by the ſecond 1.88, and the Quotient 7.44 will be the Value of the three joint Lives.

FROM the two laſt Examples it appears, that in eſtimating the Values of joint Lives, it would be an Error to ſuppoſe that they might be reduced

to an Equality, by taking a Mean Life betwixt the longeſt and ſhorteſt, for altho' 14 is a Medium betwixt 13 and 15, yet an Annuity upon thoſe three joint Lives was found to be 7.41, whereas ſuppoſing them to be each 14 Years Purchaſe, the Value is 7.44; it is true that the Difference is ſo ſmall, that it might be neglected, yet this ariſes meerly from a near Equality in the Lives, for if there had been a greater Inequality, the Concluſion would have conſiderably varied.

BEFORE I come to the fourth Problem, I think it proper to explain the Meaning of ſome Notations which I make uſe of, in order to be as clear and conciſe as I can.

I denote the Values of an Annuity upon two joint Lives, whoſe ſingle Values are M and P by $\overline{MP}$, which ought carefully to be diſtinguiſhed from the Notation MP; this laſt denoting barely the Product of one Value multiplied by the other, whereas $\overline{MP}$

$\overline{MP}$ ſtands for what was denoted in our ſecond Problem by $\frac{MP}{M+P-r-1MP}$.

IN the ſame manner, the Value of an Annuity upon the three joint Lives whoſe ſingle Values are M, P, Q, is denoted by $\overline{MPQ}$, which is equivalent to what has been expreſſed in the third Problem by $\frac{MPQ}{MP+MQ+PQ-2dMPQ}$.

THIS being premiſed, I proceed to the fourth Problem.

PROBLEM IV.

The Values of two ſingle Lives being given, to find the Value of an Annuity upon the longeſt of them, that is, to continue ſo long as either of them is in being.

SOLUTION.

LET M be the Value of one Life, P the Value of the other, $\overline{MP}$ the Value of the two joint Lives, then the Value

Value of the longeſt of the two Lives will be $M+P-\overline{MP}$. In Words thus;

From the Sum of the Values of the ſingle Lives, ſubtract the Value of the joint Lives, and the Remainder will be the Value of the longeſt.

LET us ſuppoſe two Lives, one worth 13 Years Purchaſe, the other 14, and Intereſt at 4 *per Cent.* The Sum of the Values of the Lives is 27, the Value of the two joint Lives by the Rules before given, will be found 9.23. Now, ſubtracting 9.23 from 27, the Remainder 17.77 is the Value of the longeſt of the two Lives.

IF the two Lives are equal, the Operation will be ſomething ſhorter.

BUT it is proper to obſerve in this place, that ſeveral equal Lives are concerned in an Annuity, I commonly denote one ſingle Life by M', two joint Lives by M'', three joint Lives by M''', and ſo on; ſo that the Rule for an Annuity

nuity to be granted till ſuch Time as either of the equal Lives is in being may be expreſſed by $2M'-M''$.

PROBLEM V.

The Values of three ſingle Lives being given, to find the Value of an Annuity upon the longeſt of them.

SOLUTION.

Let M, P, Q, be the Values of the ſingle Lives, $\overline{MP}$, $\overline{MQ}$, $\overline{PQ}$, the Values of all the joint Lives combined two and two, $\overline{MPQ}$ the Value of the three joint Lives, then the Value of an Annuity upon the longeſt of them is $M+P+Q-\overline{MP}-\overline{MQ}-\overline{PQ}+\overline{MPQ}$, in Words thus;

Take the Sum of the three ſingle Lives, from which Sum ſubtract the Sum of all the joint Lives combined two and two, then to the Remainder add the Value of the three joint Lives, and the Reſult will be the Value of the longeſt of the three Lives.

THUS

THUS ſuppoſing the ſingle Lives to be 13, 14, and 15 Years Purchaſe, the Sum of the Values will be 42; the Values of the firſt and ſecond joint Lives is 9.23, of the firſt and third 9.65, of the ſecond and third 10.18, the Sum of all which is 29.06, which being ſubtracted from the Sum of the Lives found before, *viz.* 42, the Remainder will be 12.94, to which adding the Value of the three joint Lives 7.41, the Sum 20.35 will be the Value of the longeſt of the three joint Lives.

BUT if the three Lives are equal, the Rule for the Value of the Life that remains laſt is $3M' - 3M'' + M'''$.

Of REVERSIONS.

PROBLEM VI.

Suppoſe A *is in Poſſeſſion of an Annuity, and that* B *after the Deceaſe of* A *is to have the Annuity for him, and his Heirs for ever, to find the preſent Value of the Reverſion.*

SOLU-

SOLUTION.

LET M be the Value of the Life in Poſſeſſion, r the Rate of Intereſt, then the preſent Value will be $\frac{1}{r-1} - M$, that is, *from the Value of the Perpetuity, ſubtract the Value of the Life in Poſſeſſion, and the Remainder will be the Value of the Reverſion.*

THUS, Suppoſing that A is 50 Years of Age, an Annuity upon his Life, Intereſt at 5 *per Cent.* would be 8.39, which being ſubtracted from the Perpetuity 20, the Remainder will be 11.61, which is the Preſent Value of the Expectation of B.

IN the ſame manner, ſuppoſing that C were to have an Annuity for him and his Heirs for ever, after the Lives of A and B, then from the Perpetuity Subtracting the Value of the longeſt of the two Lives of A and B, the Remainder will expreſs the Value of C's Expectation.

THUS, Suppoſing the Ages of A and B be 40 and 50, the Value of an Annuity upon the longeſt of theſe two

Lives would be found by the 4th Problem, to be 12.56; and this being ſubtracted from the Perpetuity 20, the Remainder is 7.44, which is the Value of *C*'s Expectation, and the Rule will be the ſame in any other caſe that may be propoſed.

PROBLEM VII.

Suppoſing that A *is in Poſſeſſion of an Annuity for his Life, and that* B *after the Life of* A, *ſhould have an Annuity for his Life only; to find the Value of the Life of* B *after the Life of* A.

THIS Caſe ought carefully to be diſtinguiſhed from the Caſe of the 6th Problem; for in that Problem, altho' the Expectant *B* ſhould die before *A*, ſtill the Heirs of *B* have the Reverſion; but in the Caſe of the preſent Problem, if *B* dies before *A*, the Heirs of *B* have no Expectation.

SOLUTION.

LET *M* be the Value of the Life of the preſent Poſſeſſor, *P* the Value of the

the Life of the Expectant, then the Value of his Expectation is $P - \overline{MP}$ In Words thus;

From the preſent Value of the Life of B, *ſubtract the preſent Value of the joint Lives of* B *and* A, *and the Remainder will be the Value of* B's *Expectation.*

THE Reaſon of which Operation is very plain, for if *B* were now to begin to receive the Annuity, it would be worth to him the Sum *P* in preſent Value; but as he is to receive nothing during the joint Lives of himſelf and *N*, the preſent Value of their two joint Lives ought to be ſubtracted from the Value of his own Life.

PROBLEM VIII.

To find the Value of one Life after two.

Thus, Suppoſe A *in Poſſeſſion of an Annuity for his Life, that* B *is to have his Life in it after* A, *and that* C *is likewiſe to have his Life in it after* B, *butſo that* B *dying before* A, C *ſucceeds* A *immediately; to find the Value of* C's *Expectation.*

SOLUTION.

LET M, P, Q, be the reſpective Values of the Lives of A, B, C, then the Value of C's Expectation is $Q - \overline{MQ} + \overline{MPQ}$, which in Words is thus expreſſed;

From the preſent Value of the Life of C, *ſubtract the Sum of the joint Lives of himſelf and* A, *and of himſelf and* B, *and to the Remainder add the Sum of the three joint Lives, and the Reſult of theſe Operations will expreſs the preſent Value of the Expectation of* C.

PROBLEM IX.

If A, B, C *agree among themſelves to buy an Annuity to be by them equally divided, whilſt they live together, then after the Deceaſe of one of them, to be equally divided between the two Survivors, then to belong entirely to the laſt Survivor for his Life; to find what each*

each of them ought to contribute towards the Purchaſe.

SOLUTION.

LET M, P, Q be the reſpective Value of the Lives of A, B, C, then what A is to contribute, is

$$M - \tfrac{1}{2}\overline{MP} + \tfrac{1}{3}\overline{MPQ} - \tfrac{1}{2}\overline{MQ}$$

What B is to contribute, is

$$P - \tfrac{1}{2}\overline{PM} + \tfrac{1}{3}\overline{MPQ} - \tfrac{1}{2}\overline{PQ}$$

What C is to contribute, is

$$Q - \tfrac{1}{2}\overline{QM} + \tfrac{1}{3}\overline{MPQ} - \tfrac{1}{2}\overline{QP}$$

In Words thus; *From the Value of the Life of* A, *ſubtract the half Sum of the Values of the joint Lives of himſelf and* B, *and of himſelf and* C, *and to the Remainder add* $\frac{1}{3}$ *of the Value of the three joint Lives, and the Sum will be what*

what A *is to contribute towards the Purchaſe.*

IN like manner, from the Value of *B*'s Life ſubtract the half Sum of the Values of the joint Lives of himſelf and *A*, and of himſelf and *C*, and to the Remainder add $\frac{1}{3}$ of the Value of the three joint Lives, and the Sum will be what *B* is to contribute.

AND again, from the Value of the Life of *C*, ſubtract the half Sum of the Values of himſelf and *A*, and of himſelf and *B*; then to the Remainder add $\frac{1}{3}$ of the Values of the three joint Lives, and this laſt Operation will ſhew what *C* is to contribute.

PROBLEM X.

Suppoſing three equal Lives of any Age given, for Inſtance 30, *and that upon the Failing of any one of them, that Life ſhall be immediately replaced, and I then receive a Sum* ſ *agreed upon, and that to perpetuity for me and my Heirs; what is the preſent Value*

lue of that Expectation, and at what Intervals of Time, one with another, may I expect to receive the ſaid Sum.

SOLUTION.

IMAGINE that there is an Annuity of 1 *l.* to be received as long as the three Lives are in being, and that the Preſent Value is M'''. which Symbol we make uſe of to repreſent the Preſent Value of an Annuity upon three equal joint Lives; now, ſince each Life is ſuppoſed to be 30 Years of Age, and that the Rate of Intereſt is 5 *per Cent.* we ſhall find, by following the Directions of the 3d Problem, that the Preſent Value of the three joint Lives is $7.64 = M'''$; this being fixed, the Preſent Value of all the Payments to be made to Eternity at equal Intervals of Time, will be $\frac{1 - dM'''}{d\,M'''} \times f$, where the Quantity d ſignifies the Intereſt of 1 *l.* In Words thus;

Multiply the Preſent Value of the three joint Lives, viz. 7.64 *by the Intereſt of*

of 1 l. *which in this Caſe is* 0.05, *and that Product, which is* 0.382, *muſt be reſerved.*

Subtract this Quantity from Unity, and the Remainder, viz. 0.618 *being divided by the Quantity reſerved, the Quotient will be* 1.62, *and this being multiplied by the Sum* ſ, *which we may ſuppoſe* 100 l. *the Product will be* 162 l. *and this is the preſent Value of all the Payments that will be made to Eternity, at equal Intervals of Time upon the failing of a Life, which is to be immediately replaced.*

As for the Intervals of Time after which thoſe Replacements will be made, they may be found thus;

Look in the ſecond of our Tables for the Number 7.64, which is the Value of the three joint Lives, and over againſt it will be found the Number anſwering, which is between 9 and and 10; and ſo it may be ſaid that the Replacements will be made at every Interval of about 9 or 10 Years.

BUT that Interval may be determined a little more accurately, by help of a Table of Logarithms, by taking the Logarithm of the Quantity $\frac{1}{1-dM''}$ and dividing it by the Logarithm of r.

THE Logarithm of $\frac{1}{1-dM''}$ is 0.2090115; the Logarithm of r is 0.0211893; and the firſt being divided by the ſecond, the Quotient is 9.86, which ſhews that the Replacements will be made at Intervals a little more than $9\frac{3}{4}$ Years.

PROBLEM XI.

Suppoſing, as before, three equal Lives of 30, *and that the Lives are not to be renewed, till after the failing of any two of them, and that a Sum* p *is then to be received, and that perpetually, after the failing of two Lives, what is the preſent Value of that Expectation.*

SOLUTION.

MAKE $3M' - 2M'' = A$, let the Intereſt of 1 *l.* be $= d$, then the preſent

ſent Value of that Expectation will be $\frac{1-dA}{dA} \times p$.

BUT to know the Intervals of Time after which the Lives will be filled up, take the Logarithm of the Quantity $\frac{1}{1-dA}$, and divide it by the Logarithm of r.

THE Value of a ſingle Life of 30, Intereſt at 5 *per Cent.* is found in our Tables to be 13 Years Purchaſe, the Value M'' of two joint Lives by Problem II. is 9.83; and the Value M''' of three joint Lives by Problem III. is 7.85; then $3M'' - 2M'''$, or the Difference between the Triple of two joint Lives, and the Double of two joint Lives will be $13.59 = A$, then $\frac{1-dA}{dA} \times p$ will be found to be $0.473p$, and the Intervals of Time will be 23.32, that is, nearly $23\frac{1}{3}$ Years.

PROBLEM XII.

Suppoſing ſtill the Lives to be 30, *and that they are not to be renewed till after the Extinction of all three, and*

and that a Sum q *is then to be received, and that perpetually after every Renewal, what is the preſent Value of that Expectation?*

SOLUTION.

MAKE $3M' - 3M'' + M''' = B$, then the preſent Value of that Expectation will be $\frac{1-dB}{dB} \times q$; here B will be found to be 17.84, and conſequently $\frac{1-dB}{dB} \times q$ will be $= 0.121 \times q$.

AND the Intervals of Time will be the Logarithm of the Quantity $\frac{1}{1-dB}$ divided by the Logarithm of r, which in this Caſe would be 45.62, that is, nearly $45\frac{3}{5}$ Years.

COROLLARY.

HENCE it will be eaſy to the Proprietor of the Lives, to find which is moſt advantageous to him, to fill up a Life as ſoon as it is vacant, or not to fill up before the Vacancy of two, or to let them all drop before the Renewal.

REMARK.

IT is not to be imagined that if Intereſt of Money was higher or lower than 5 *per Cent.* the Intervals of Time after which the Renewals are made, would be the ſame as they are now, for it will be found, that as Intereſt is higher, the Intervals will be ſhorter; and as it is lower, ſo the Intervals will be longer; yet one might make it an Objection to our Rules, that the length of Life would thereby ſeem to depend upon the Rate of Intereſt. To anſwer this Difficulty, it muſt be obſerved, that the calculating of Time imports no more, than that conſidering the Circumſtances of the Purchaſer and the Proprietor of the Lives, in reſpect to the Rate of Intereſt agreed upon, and the Sum to be given upon the Renewal of a Life, or Lives, the Proprietor makes the ſame Advantage of his Money, as if he had agreed with the Purchaſer, that he ſhould pay him a certain Sum of Money at equal Intervals

vals of Time, for redeeming the Riſque which he the Purchaſer runs of paying that Sum when the Life or Lives drop, but the real Intervals of Time will be ſhewn afterwards.

ALTHO' it ſeldom happens that in Contracts about Lives, any more than three are concern'd, yet I hope it will not be diſpleaſing to our Readers to have this Speculation carry'd a little farther.

BUT as general Rules are beſt inculcated by particular Examples, I ſhall take the Caſe of five Lives, and expreſs the ſeveral Circumſtances of them in ſuch manner, as that they may be a ſure Guide to all other Caſes of the ſame kind, let the Number of Lives be what it will; let therefore the following Expreſſions be written,

$$M^{IV}$$
$$5M''' - 5M^{IV}$$
$$10M'' - 15M''' + 6M^{IV}$$
$$10M' - 20M'' + 15M''' - 4M^{IV}$$
$$5M - 10M' + 10M'' - 5M''' + 1M^{IV}$$

The

THE firſt Term M''''' repreſents properly the preſent Value of an Annuity upon five equal joint Lives, but from thence may be deduced the Time of their joint continuance, or the Time in which it may be expected that one of them will fail, it being as I have ſaid before, the Logarithm of $\frac{1}{1-dM'''''}$ divided by the Logarithm of r, however for ſhortneſs ſake, I call for the preſent that Expreſſion the Time.

THE two next Terms $5M''''-4M'''''$ repreſents the Time in which two of the Lives will fail.

THE three next Terms $10M'''-15M''''+6M'''''$, repreſent the Time in which three out of the five Lives will fail.

THE four next $10M''-20M'''+15M''''-4M'''''$, repreſent the Time in which four out of the five Lives will fail.

THE five next $5M'-10M''+10M'''-5M''''+1M'''''$, repreſent the Time

Time in which all the five Lives will be extinct.

Now the Law of the Generation of the Co-efficients is thus.

1°. TAKE all the Terms which are affected with the Mark M''', beginning from the uppermoſt, the Co-efficients 1—4+6—4+1, which are the Terms of the Binomial 1—1, rais'd to the fourth Power, which is leſs by one, than the Number of Lives concerned.

2°. TAKE the Terms which are affected with the Mark M''', and prefix to them in order, the Product of the Number 5 by the Co-efficients 1—3+3—1, which are the Terms of the Binomial 1—1 raiſed to its Cube, that is, to a Power leſs by two than the Number of Lives concerned.

3°. TAKE all the Terms which are affected with the Mark M'', and prefix to them in order, the Product of the Number 10, multiplied by the Co-efficients 1—2+1, which are the Terms of the Binomial 1—1 raiſed to its Cube, that is, to a Power leſs by three,

three, than the Number of Lives concerned.

4°. TAKE all the Terms which are affected with the Mark M', and prefix to them the Product of the Number 10, multiplied by the Terms of the Binomial $1-1$, raiſed to the Power whoſe Index is 1, that is to a Power leſs by four, than the Number of Lives concerned.

5°. TAKE all the Terms which are affected with the Mark M', and prefix to them the Product of the Number 5, multiplied by the Binomial $1-1$, raiſed to a Power leſs by 5 than the Number of Lives concerned, which in this Caſe happening to be nothing, or 0, degenerates barely into Unity.

As for the Multiplicators, conceiving that the Multiplicator of the firſt Term M'''' is 1, all the Multiplicators will be 1, 5, 10, 10, 5, which are all, except the laſt, the Co-efficients of the Binomial $1+1$, raiſed to its fifth Power, that is, to a Power equalling the Number of all the Lives.

N. B.

N. B. THE Exception here given, does not fall upon the Number 5, but upon the laſt Term of the fifth Power, 1+5+10+10+5+1, which laſt 1, is rejected.

Of SUCCESSIVE LIVES.

PROBLEM XI.

If A *enjoys an Annuity for his Life, and at his Deceaſe has the Nomination of a Succeſſor* B, *who is alſo to enjoy the Annuity for his Life, to find the preſent Value of the two ſucceſſive Lives.*

SOLUTION.

LET the Values of the Lives be M and P; let d be the Intereſt of 1 *l.* then the Value of the two ſucceſſive Lives will be $M+P-dMP$.

BUT if the Succeſſor B was himſelf to have the Nomination of a Life $\mathfrak{Q}$; then the Value of the three ſucceſſive Lives would be $M+P+\mathfrak{Q}-d \times \overline{MP+M\mathfrak{Q}+P\mathfrak{Q}+dd} \times MP\mathfrak{Q}$.

BUT before I proceed, it is proper to obſerve that the Expreſſions MP, $M\mathcal{Q}$, $P\mathcal{Q}$ and $MP\mathcal{Q}$, ſignify barely Products, which is conformable to the uſual Algebraic Notation; this I take notice of, for fear thoſe Expreſſions ſhould be confounded with others that I have made uſe of before, *viz.* $\overline{MP}$, $\overline{M\mathcal{Q}}$, $\overline{P\mathcal{Q}}$ and $\overline{MP\mathcal{Q}}$, which denoted joint Lives.

BUT to compriſe under one general Rule all the poſſible Caſes that may happen about any Number of ſucceſſive Lives, it will be proper to expreſs it in Words at length, thus;

From the Sum of all the Lives, ſubtract the Sum of the Products of all the Lives combined two and two, which Sum of Products before they are ſubtracted, ought to be multiplied by the Intereſt of 1 l.

To this add the Sum of the Products of all the Lives taken three and three, but multiplied again by the Square of the Intereſt of 1 l.

From

From this ſubtract the Sum of the Products of all the Lives taken four and four, but multiplied again by the Cube of the Intereſt of 1 l. *and ſo on by alternate Additions and Subtractions, ſtill obſerving that if there was occaſion to take the Lives five and five, ſix and ſix,* &c. *the Intereſt of* 1 l. *ought to be raiſed to the* 4*th Power, and to the* 5*th, and ſo on.*

BUT all thoſe Operations would be very much contracted, if the Lives to be nominated were always of the ſame Age; for Inſtance 30, for ſuppoſe *M* to be the Value of an Annuity on an Age of 30, and *d* to be the Intereſt of 1 *l.* then the preſent Value of all the ſucceſſive Lives, of which the Number is *n*, would be $\frac{1-\overline{1-dM}^{n}}{d}$. In Words thus;

Multiply the Value of one Life by the Intereſt of 1 l. *let the Product be ſubtracted from Unity, and let the Remainder be raiſed to that Power which anſwers to the Number of Lives; then*

this Power being again ſubtracted from Unity, let the Remainder be divided by the Intereſt of 1 l. *and the Quotient will be the preſent Value of all the ſucceſſive equal Lives.*

And again, if the Number of thoſe Lives were infinite, the Sum would barely be $\frac{1}{d}$.

PROBLEM XII.

Of a Perpetual Advowſon.

1°. I ſuppoſe that at the Time of the Demiſe of the Incumbent, the Patron would receive the Sum *ſ*, for alienating his Right of the next Preſentation, if the Law did not forbid the Alienation in that Circumſtance of Time.

2°. I ſuppoſe that when this Right is transfer'd, the Age of the Incumbent is ſuch, that an Annuity upon his Life would be worth *M* Years Purchaſe, when the Intereſt of 1 *l.* is *d.*

THIS

THIS being ſuppoſed, the Right of the next Preſentation is worth $\overline{1-dM} \times f$, and the Right of Patronage or perpetual Recurrency of the like Circumſtances to Eternity would be worth $\frac{1-dM}{dM} \times f$. In Words thus;

Take the preſent Value of the Life of the Incumbent, and multiply it by the Intereſt of 1 l. *and reſerve the Product.*

Subtract this Product from Unity, and let the Remainder be multiplied by the Sum expected f, *and the new Product will ſhew the Right of the next Preſentation; let alſo this be reſerved.*

Then divide the ſecond Quantity reſerved by the firſt, and the Quotient will ſhew the preſent Value of the Right of Patronage, or perpetual Recurrency.

THUS, ſuppoſing the Life of the Incumbent worth 8 Years Purchaſe, the Rate of Intereſt 5 *per Cent.* and the Sum *f* to be 100 *l.* the Right of the next Preſentation would be worth 60 *l.*

60 *l.* and the Right of perpetual Recurrency 150 *l.*

PROBLEM XIII.

Of a Copy-hold.

Suppoſing that every Copy-hold Tenant pay to the Lord of the Manor a certain Fine on Admittance, and that every Succeſſor does the like; to find the Value of the Copy-hold computed from the Time of a Fine being paid, independently from the Fine that may be given on Alienation.

SOLUTION.

I ſuppoſe that the Value of the Life of the preſent Tenant, and the Life of every future Succeſſor when he comes to Poſſeſſion is the ſame; this being admitted, let M be the Value of a Life, d the Intereſt of 1 *l.* and f the Fine to be paid, then the preſent Value of the Copy-hold will be $\frac{1-dM}{dM} \times f$, and this Expreſſion being exactly the ſame

ſame as that whereby the Right of Patronage has been determined, needs no Explanation in Words.

ONLY it is neceſſary to obſerve, that the Sum f paid in Hand being added to this, will make the Canon ſhorter, and will be reduced to $\frac{s}{dM}$, which may be expreſſed thus in Words.

Divide the Fine by the Product of the Life, multiplied by the Intereſt of 1 l.

THUS, if the Life of a Tenant is worth 12 Years Purchaſe, and the Fine to be paid on Admittance 56 *l.* and alſo the Rate of Intereſt 5 *per Cent.* then the preſent Value of the Copyhold is $93\frac{1}{3}$ *l.*

PROBLEM XIV.

A *borrows a certain Sum of Money, and gives Security that it ſhall be repaid at his Deceaſe with the Intereſts, to fix the Sum which is then to be paid.*

SOLU-

SOLUTION.

LET the Sum borrowed be s, the Life of the Borrower M Years purchaſe, d the Intereſt of 1 *l.* then the Sum to be paid at *A*'s Deceaſe will be $\frac{s}{1-dM}$; thus, ſuppoſing $s = 800$, $M = 11.83$, $d = 0.05$, then $\frac{s}{1-dM}$ would be found $= 1958$ *l*; in the ſame manner, if the Sum to be paid at *A*'s Deceaſe, was to be an Equivalent for his Life, unpaid at the Time of the Purchaſe, that Sum would be $\frac{M}{1-dM} = 2946$ *l.* Suppoſing the Annuity received to be 100 *l.* as alſo the Life of *A* 11.83 Years Purchaſe.

PROBLEM XV.

A *borrows a Sum* s, *payable at his Deceaſe, but with this Condition, that if he dies before* B, *then the whole Sum is to be loſt to the Lender; to find what* A *ought to pay at his Deceaſe in caſe he ſurvives* B.

SOLU-

SOLUTION.

LET us ſuppoſe, as before, that *A* is 40 Years of Age, that the Sum borrowed is 800 *l.* and that Intereſt of Money is 5 *per Cent.* Farther, let it be ſuppoſed that *B* is 70 Years of Age, then, 1°. determine what *A* ſhould pay at his Deceaſe, if the Life of *B* was not concerned; by the Solution of the preceding Problem, we find the Sum to be 2316 *l.* But we ought to conſider that the Lender having a Chance to loſe his Money, there ought to be a Compenſation for the Riſque he runs, which is founded on the poſſibility of a Man of ſeventy outliving a Man of forty. Now, by the Rules to be delivered in the next Problem, we ſhall find that the Probability of that Contingency is meaſured by the Fraction $\frac{4}{23}$, and therefore the Probability of the youngeſt Life's ſurviving the oldeſt is $\frac{19}{23}$. Now this being the Meaſure of the Proba-

bility which the Lender has of being repaid, the Sum 2316 ought to be increaſed in the proportion of 23 to 19, which will make it to be 2804 *l.* nearly.

Of the Probabilities of Survivorſhip.

PROBLEM XVI.

Any Number of Lives being given, to find their Probability of Survivorſhip.

SOLUTION.

LET *A*, *B*, *C*, *D*, &c. be the Lives, whereof *A* is ſuppoſed to be the youngeſt, *B* the next to it, *C* the next, &c. and ſo the laſt the oldeſt.

LET *n*, *p*, *q*, *s*, *t*, &c. be the reſpective Intervals intercepted between the Ages of thoſe Lives, and the Extremity of old Age ſuppoſed at 86; then the Probabilities of any one of thoſe Lives ſurviving all the reſt, will be

for

$$\begin{array}{l|l}
\text{for } A & 1 - \frac{p}{2n} - \frac{qq}{6np} - \frac{s^3}{12npq} - \frac{t^4}{20npqs} \\
B & + \frac{p}{2n} - \frac{qq}{6np} - \frac{s^3}{12npq} - \frac{t^4}{20npqs} \\
C & + \frac{qq}{3np} - \frac{s^3}{12npq} - \frac{t^4}{20npqs} \\
D & + \frac{s^3}{4npq} - \frac{t^4}{20npqs} \\
E & + \frac{t^4}{5npqs} \\
\&c. &
\end{array}$$

Here ſome few things may be obſerved.

1°. THAT the Probability of the youngeſt Life ſurviving all the reſt, always begins with Unity, and that it is expreſſed by ſo many Terms as there are Lives concerned.

2°. THAT the Probabilities of the other Lives ſurviving all the reſt, are always expreſſed each by one Term leſs than the preceding.

3°. THAT each firſt Term of thoſe whereby each Probability is expreſſed, is always the Sum of all the other Terms ſtanding above it.

4°. THAT the Numbers 2, 6, 12, 20, 30, *&c.* made uſe of in the Denominators of the Fractions are generated by the Multiplication of the following Number, 1 × 2, 2 × 3, 3 × 4, 4 × 5, *&c.* It would take up too much room to explain this general Rule in Words at length, for which Reaſon I ſhall content my ſelf with explaining only the Caſes of two and three Lives, which are the moſt neceſſary.

And, Firſt, *if there be two Lives of a given Age, ſuch as* 40 *and* 70, *take their Complements of Life, which as I have explained before, are the Differences between* 86 *and the reſpective Ages, thoſe Complements therefore are* 46 *and* 16.

Divide the ſhorteſt Complement by the Double of the Longeſt, and the Quotient will expreſs the Probability of the oldeſt Life ſurviving the youngeſt.

THUS in the preſent Caſe, the ſhorteſt Complement being 16, and the

the double of the longeſt being 92, I divide 16 by 92, and the Quotient $\frac{16}{92}$ or $\frac{4}{23}$ will expreſs the Probability required.

SUBTRACT this Fraction from Unity, and the Remainder $\frac{19}{23}$ will expreſs the Probability of the youngeſt Life ſurviving the oldeſt.

SO that the Odds of the youngeſt Life ſurviving the oldeſt, are 19 to 4.

THE Caſe of three Lives is thus, Suppoſe there are three Lives of a given Age, ſuch as 40, 45, and 60; take their reſpective Complements of Life, which are 46, 41, 26, then divide the Square of the ſhorteſt Complement by 3 times the Product of the other two, and the Quotient will expreſs the Probability of the oldeſt Life ſurviving the other two.

DIVIDE the middlemoſt Complement by the Double of the greateſt, and from the Quotient ſubtract the Square of the leaſt by 6 Times the Product of the other two, and the Remainder will expreſs the Probability of the

the middlemoſt Life ſurviving the other two.

SUBTRACT the Sum of the two foregoing Probabilities from Unity, and the Remainder will expreſs the Probability of the youngeſt Life ſurviving the other two.

THUS in the Caſe propoſed, the Probability of the oldeſt Life ſurviving the other two, will be found $\frac{676}{5658} = \frac{3}{25}$ nearly.

THE Probability of the middlemoſt Life ſurviving the other two will be $\frac{4367}{11316} = \frac{5}{13}$ nearly.

The Probability of the youngeſt Life ſurviving the other two will be $\frac{3}{5}$ nearly.

PROBLEM XVIII.

Any Number of Lives being given, to find the Probability of the Order of their Survivorſhip.

SOLUTION.

SUPPOSE the three Lives to be thoſe of *A*, *B*, *C*, and that it be required to aſſign

aſſign the Probability of Survivorſhip as limited to the Order in which they are written, ſo that *A* ſhall both ſurvive *B* and *C*, and *B* alſo ſurvive *C*. This being ſuppoſed, let n, p, q repreſent the reſpective Complements of Life, of the youngeſt, middlemoſt, and oldeſt, then the Probabilities of the ſix different Orders that there are in three things, will be as follows;

A, *B*, *C*	$1 - \frac{p}{2n} - \frac{q}{2p} + \frac{qq}{6np}$
A, *C*, *B*	$- \quad - \quad - \quad - \frac{q}{2p} - \frac{qq}{3np}$
B, *A*, *C*	$\frac{p}{2n} - \frac{q}{2n} + \frac{qq}{6np}$
B, *C*, *A*	$\frac{q}{2n} - \frac{qq}{3np}$
C, *A*, *B*	$\frac{qq}{6np}$
C, *B*, *A*	$\frac{qq}{6np}$

In Words thus;

1°. *Divide the middlemoſt Complement by the double of the greateſt, and let the Quotient be ſubtracted from Unity.*

2.

2°. *From that Remainder ſubtract again the Quotient of the ſhorteſt Complement divided by the Double of the Middlemoſt.*

3°. *To that new Remainder add the Quotient ariſing from the Square of the ſhorteſt Complement divided by ſix times the Product of the greateſt and middlemoſt multiplied together, and this laſt Sum will expreſs the Probability of the firſt Order.*

THE Probability of the Second will be found thus;

1°. *Divide the ſhorteſt Complement by the double of the middlemoſt, and reſerve the Quotient.*

2°. *Divide the Square of the ſhorteſt by three times the Product of the longeſt Complement, multiplied by the Middlemoſt, and reſerve the new Quotient.*

3°. *Let the ſecond Quotient be ſubtracted from the firſt, and the Remainder will expreſs the Probability of the happening of the ſecond Order.*

THE Probability of the third Order will be found as follows.

1°. *Divide the middlemoſt Complement by the Double of the Greateſt, and reſerve the Quotient.*

2°. *Divide the ſhorteſt Complement by the double of the longeſt, and reſerve the Quotient.*

3°. *Divide the Square of the ſhorteſt Complement by ſix times the Product of the longeſt and middlemoſt multiplied together, and reſerve the Quotient.*

4°. *From the firſt Quotient reſerved, ſubtract the ſecond; then to the Remainder add the Third, and the Reſult of theſe Operations will expreſs the Probability of the third Order.*

THE Probability of the fourth Order will be found thus.

1°. *Divide the ſhorteſt Complement by the double of the longeſt, and reſerve the Quotient.*

2°. *Divide the Square of the ſhorteſt Complement by three Times the Product of the longeſt and middlemoſt, and reſerve the new Quotient.*

3°. *From the firſt Quotient reſerved, ſubtract the ſecond, and the Remainder will expreſs the Probability of the fourth Order.*

THE fifth Order will be found as follows.

Divide the Square of the ſhorteſt Complement by ſix times the Product of the longeſt and middlemoſt, multiplied together, and the Quotient will expreſs the Probability required.

THE Probability of the laſt Order is the ſame as that of the fifth.

PROBLEM XVII.

D, *whilſt in Health, makes a Will, whereby he bequeaths* 500 l. *to* E, *and* 300 l. *to* F. *with this Condition, that if either of them dies before him, the whole is to go to the Survivor of the two; what are the Values of the Expectations of* E *and* F, *eſtimated from the time that the Will was writ.*

SOLU-

SOLUTION.

SUPPOSE D to be 70 Years of Age, E 36, and F 45; ſuppoſe alſo that d repreſents the Intereſt of 1 *l.* when Intereſt is at 5 *per Cent.*

AN Annuity upon the Life of D is worth 5.77, as appears from our Tables, which Value we may call M.

WHEREFORE if it was ſure that D would die before either of them, the Expectation of E upon that Account, would be worth in preſent Value $\overline{1-dM} \times 500$, and the Expectation of F, $\overline{1-dM} \times 300$; which being reduced to Numbers, are reſpectively 355 *l.* 15 *s.* and 213 *l.* 9 *s.*

BUT as this depends on the Probability of D's dying firſt, we are to look for that Probability, which is compoſed of two Parts, that is, when the Order of Survivorſhip is either E, F, D, or F, E, D; now the Order E, F, D, is the ſame as A, B, C, in the preceding Problem, whereof the Probability is $1-\frac{p}{2n}-\frac{q}{2p}+\frac{qq}{6np}$, and the Or-

3°. *From the firſt Quotient reſerved, ſubtract the ſecond, and the Remainder will expreſs the Probability of the fourth Order.*

THE fifth Order will be found as follows.

Divide the Square of the ſhorteſt Complement by ſix times the Product of the longeſt and middlemoſt, multiplied together, and the Quotient will expreſs the Probability required.

THE Probability of the laſt Order is the ſame as that of the fifth.

PROBLEM XVII.

D, *whilſt in Health, makes a Will, whereby he bequeaths* 500 l. *to* E, *and* 300 l. *to* F. *with this Condition, that if either of them dies before him, the whole is to go to the Survivor of the two; what are the Values of the Expectations of* E *and* F, *eſtimated from the time that the Will was writ.*

SOLU-

SOLUTION.

SUPPOSE *D* to be 70 Years of Age, *E* 36, and *F* 45; ſuppoſe alſo that *d* repreſents the Intereſt of 1 *l.* when Intereſt is at 5 *per Cent.*

AN Annuity upon the Life of *D* is worth 5.77, as appears from our Tables, which Value we may call *M.*

WHEREFORE if it was ſure that *D* would die before either of them, the Expectation of *E* upon that Account, would be worth in preſent Value $\overline{1-dM}\times 500$, and the Expectation of *F*, $\overline{1-dM}\times 300$; which being reduced to Numbers, are reſpectively 355 *l.* 15 *s.* and 213 *l.* 9 *s.*

BUT as this depends on the Probability of *D*'s dying firſt, we are to look for that Probability, which is compoſed of two Parts, that is, when the Order of Survivorſhip is either *E*, *F*, *D*, or *F*, *E*, *D*; now the Order *E*, *F*, *D*, is the ſame as *A*, *B*, *C*, in the preceding Problem, whereof the Probability is $1-\frac{p}{2n}-\frac{q}{2p}+\frac{qq}{6np}$, and the Or-

Age may juſtly expect to continue in being.

I have found by a Calculation deduced from the Method of Fluxions, that upon Suppoſition of an equable Decrement of Life, the Expectation of Life would be expreſſed by $\frac{1}{2}n$, ſuppoſing n to denote its Complement.

HOWEVER, if that Interval be once attained, there ariſes a new Expectation of $\frac{1}{4}n$, and afterwards of $\frac{1}{8}n$, and ſo on. This being laid down, I ſhall proceed farther.

PROBLEM XVIII.

To find the Expectation of two joint Lives, that is, the Time which two Lives may expect to continue together in being.

SOLUTION.

LET the Complements of the Lives be n and p, whereof n be the longeſt and

and p the ſhorteſt, then the Expectation of the two joint Lives will be $\frac{1}{2}p - \frac{pp}{6n}$, in Words thus.

From $\frac{1}{2}$ the ſhorteſt Complement, ſubtract the 6th Part of his Square, divided by the greateſt, the Remainder will expreſs the Number of Years ſought.

THUS, ſuppoſing a Life of 40, and another of 50, the ſhorteſt Complement will be 36, the greateſt 46, $\frac{1}{2}$ of the ſhorteſt will be 18, the Square of 36 is 1236, whereof the ſixth Part is 216, which being divided by 46, the Quotient will be $\frac{216}{46} = 4.69$ nearly; and this being ſubtracted from 18, the Remainder 13.31 will expreſs the Number of Years due to the two joint Lives.

COROLLARY.

IF the two Lives be equal, the Expectation of the two joint Lives will be

be $\frac{1}{3}$ part of their common Complement.

PROBLEM XIX.

Any Number of Lives being given, whether equal or unequal, to find how many Years they may be expected to continue together.

SOLUTION.

1°. TAKE $\frac{1}{2}$ of the ſhorteſt Complement.

2°. TAKE $\frac{1}{6}$ part of the Square of the ſhorteſt, which divide ſucceſſively by all the other Complements, then add all the Quotients together.

3°. TAKE $\frac{1}{12}$ part of the Cube of the ſhorteſt Complement, which divide ſucceſſively by the Product of all the other Complements, taken two and two.

4°. THEN take $\frac{1}{20}$ part of the Biquadrate of the ſhorteſt Complement, which

which divide ſucceſſively by the Products of all the other Complements, taken three and three, and ſo on.

5°. THEN from the Reſult of the firſt Operation, ſubtract the Reſult of the ſecond, to the Remainder add the Reſult of the third, from the Sum ſubtract the Reſult of the fourth, and ſo on.

6°. THE laſt Quantity remaining after theſe alternate Subtractions and Additions, will be the thing required.

N. B. The Diviſors 2, 6, 12, 20, &c. are the Products of 1 by 2, of 2 by 3, of 3 by 4, of 4 by 5, &c.

COROLLARY.

IF all the Lives be equal, add Unity to the Number of Lives, and divide their common Complement by that Number thus increaſed by Unity, and the Quotient will always expreſs the Time due to their joint Continuance.

PROBLEM XX.

Two Lives being given, to find the Number of Years due to the longeſt.

SOLUTION.

FROM the Sum of the Years due to each Life, ſubtract the Number of Years due to their joint Continuance, the Remainder will be the Number of Years due to the Longeſt, or Survivor of them both.

THUS, ſuppoſing a Life of 40, and another of 50, the Number of Years due to the Life of 40, is 23; the Number of Years due to the Life of 50, is 18; from the Sum of 23 and 18, *viz.* 41, ſubtract 13.31 due to their joint Continuance, the Remainder 27.69 will be the Time due to the longeſt.

COROLLARY.

IF the Lives be equal then $\frac{2}{3}$ of their common Complement will be the Number

ber of Years due to the Survivor.

THUS, ſuppoſing two Lives of 50, then their Complement will be 36; whereof two thirds will be 24; which is the Time due to the Survivor of the two.

PROBLEM XXI.

Any Number of Lives being given, to find the Number of Years due to the Longeſt.

SOLUTION.

LET the Years due to each Life be reſpectively denoted by *M*, *P*, *Q*, *S*, &c. then let the joint Lives taken, two and two be denoted by $\overline{MP}$, $\overline{MQ}$, $\overline{MS}$, $\overline{PQ}$, &c. let alſo the joint Lives, taken three and three, be denoted by $\overline{MPQ}$, $\overline{MPS}$, $\overline{MQS}$, $\overline{PQS}$, &c. Moreover, let the joint Lives taken four and four, be denoted by $\overline{MPQS}$, &c. then if there be three Lives, the Time due to the longeſt will be

$$M-\overline{MP}+\overline{MPQ}$$
$$+P-\overline{MQ}$$
$$+Q-\overline{PQ}$$

BUT if all the Lives be equal, let n be their common Complement, then the Time due to the longeſt, will be $\frac{3}{4}n$.

IF there be four Lives, the Time due to the longeſt will be

$$M-\overline{MP}+\overline{MPQ}-\overline{MPQS}$$
$$+P-\overline{MQ}+\overline{MPS}$$
$$+Q-\overline{MS}+\overline{MQS}$$
$$+S-\overline{PQ}+\overline{PQS}$$
$$-\overline{PS}$$
$$-\overline{QS}$$

BUT if all the Lives be equal, the Time due to the longeſt will be expreſſed by $\frac{4}{5}$ of their common Complement.

Univerſally, if the Common Complement of equal Lives be n, and the Number of Lives p, the Number of Years

Years due to the Longeſt of them will be $\frac{p}{p+1} \times n$.

PROBLEM XXII.

Any Number of equal Lives being given, to find the Time in which one, or two, or three, &c. *of them will fail.*

SOLUTION.

LET n be their common Complement, p the Number of all the Lives, q the Number of thoſe which are to fail, then $\frac{q}{p+1} \times n$ will expreſs the Time required. In Words thus;

Multiply the common Complement of the Lives by the Number of the Lives that are to drop, and divide the Product by the Number of all the Lives increaſed by Unity.

THUS, ſuppoſing 100 Lives, each of 40 Years of Age, it will be found that 5 of them will drop in about two Years and a Quarter.

BUT

BUT if we put t for the Time given we ſhall have the four following Equations;

1°. $t = \frac{q^n}{p+1}$

2°. $q = \sqrt[n]{p+1 \times t}$

3°. $p = \frac{n\,q - t}{t}$

4°. $n = \frac{\log. p+1 \times t}{\log. q}$

In which any three of the four Quantities n, p, q, t, being given, the fourth will be known.

THIS Speculation might be carried to any Number of unequal Lives, but my Deſign not being to perplex the Reader with too great Difficulties, I ſhall forbear at preſent to proſecute the thing any farther.

PRO-

PROBLEM XXIII.

A *who is* 30 *Years of Age, buys an Annuity for a limited Time of his Life, ſuppoſe* 10 *Years, on Condition that if he dies before the Expiration of that Time, the Purchaſe-Money is wholly to be loſt to his Heirs; to find the preſent Value of the Purchaſe, ſuppoſing Intereſt at* 5 per Cent.

SOLUTION.

LET *M* be the Value of an Annuity for a Life of 30, *P* the Value of an Annuity for a Life of 40, which exceeds the Life given by the limited time of ten Years; let *Q* be the Value of an Annuity certain for 10 Years, *n* the Complement of Life at the beginning of the limited time of 10 Years, and *p* the Complement of Life at the End of that Time; then the preſent

preſent Value of the Purchaſe will be $M - \frac{p}{n} P \times \overline{1 - dQ}$; d being the Intereſt of 1 *l.* In words thus:

1°. *Take from the Tables the Value of an Annuity certain for the limited Time given at the given Rate.*

2°. *Multiply this Value by the Intereſt of* 1 *l. and ſubtract the Product from Unity, and reſerve the Remainder.*

3° *Multiply the Value of an Annuity for an Age, exceeding the Age given by the limited Time* 10, *which in this Caſe is an Annuity for a Life of* 40, *by the Complement of Life at that Time, in this Caſe* 46; *and divide the Product by the Complement of Life at the beginning of the limited Time here,* 56; *reſerving the Quotient.*

4°. *Multiply the two Numbers reſerved together, and ſubtract the Product from the Value of an Annuity for the given Age, here* 30; *and the Remainder will be the Value of the Thing required.*

IT

IT is evident by the foregoing Proceſs, that altho' the Queſtion there propoſed is particular, yet the Solution is general, which Method often practiſed in my *Doctrine of Chances*, is of ſingular Uſe to fix the Reader's Imagination.

PROBLEM XXIV.

A *pays an Annuity of* 100 l. *during the Lives of* B *and* C, *each* 34 *Years of Age*; *to find what* A *ought to give in preſent Money to buy off the Life of* B, *ſuppoſing Intereſt at* 4 per Cent.

SOLUTION.

IT will be found by our Tables that an Annuity upon a Life of 34 is worth 14.12 Years Purchaſe, and by the Rules before delivered, that an Annuity upon the longeſt of the two Lives of *B* and *C* is worth 18.40; hence it is very plain, that to buy off the Life of *B*, *A* muſt pay the Difference between 18.40 and 14.12, which being 4.28, it follows that *A* ought to pay 428 *l.*

IN the ſame manner, if *A* were to pay an Annuity during the three Lives of *B*, *C*, *D*, whether of the ſame or different Ages, it would be eaſy to determine what *A* ought to pay to buy off one of the Lives of *B*, *C*, *D*, or any two of them, or to redeem the whole.

FOR, 1°. if the Life of *D* is to be bought off, then from the Value of the three Lives ſubtract the Value of the two Lives of *B* and *C*, and the Remainder is what is to be given to buy off the Life of *D*.

2°. IF the two Lives of *C* and *D* were to be bought off, then from the Value of the three Lives, ſubtract the Life of *B*, and the Remainder is what is to be given to buy off thoſe two Lives.

Laſtly, It is plain that to redeem the whole, the Value of the three Lives ought to be paid.

PROBLEM XXV.

A whoſe Life is worth 14 *Years Purchaſe, ſuppoſing Intereſt at* 4 per Cent.

is to enjoy an Annuity of 100 l. *during the Term of* 31 *Years*; B *has the Reverſion of it after the Deceaſe of* A *for the Term remaining*; *to find the Value of* B's *Expectation.*

SOLUTION.

SINCE the Life of *A* is ſuppoſed to be worth 14 Years Purchaſe when Intereſt is at 4 *per Cent.* it follows from the Tables that *A* muſt be about 35 Years of Age, therefore find by the twenty-third Propoſition the Value of an Annuity on a Life of 35, to continue the limited Time of 31 Years; let that Value be ſubtracted from the Value of an Annuity certain, to continue 31 Years; and the Remainder will be the Value of the Reverſion.

PROBLEM XXVI.

A *is to have an Annuity of* 100 l. *for him and his Heirs after the failing of any one of the Lives* M, P, Q, *the firſt of which is worth* 13 *Years Purchaſe, the ſecond* 14, *and the third* 15; *to find the preſent Value of his Expecta-*

tion, Intereſt of Money being ſuppoſed at 4 per Cent.

SOLUTION.

BY the Example to Prob. 3. it appears, that an Annuity upon the above 3 joint Lives is worth 7.41 Years Purchaſe; let this be ſuppoſed $=R$, and let f repreſent the preſent Value of a Perpetuity of 100 *l.* which in this Caſe is 2500 *l.* then the preſent Expectation of A will be worth $\overline{1-\frac{1}{2}R} \times f$. In Words thus;

Multiply the Value of the three joint Lives by the Intereſt of 1 *l. then ſubtracting that Product from Unity, let the Remainder be multiplied by the Value of the Perpetuity, and the Product will be the Expectation required.*

IN this Caſe 7.41, multiplied by 0.04, produces 0.2964, and this Product ſubtracted from Unity, leaves 0.7036; now this Remainder being multiplied by 2500, produces 1759 *l.* the Expectation of A.

BUT if the Problem had been, that A ſhould not have the Annuity before the

the Failing of any two of thoſe Lives, then from the Sum of all the joint Lives combined two and two, ſubtract the double Value of the three joint Lives, and let the Remainder be called T, then the Expectation of A will be worth $\overline{1-dT}\times f$; now, by the Rules before delivered, we ſhall find that the Sum of all the joint Lives combined two and two, is 29.06, from which ſubtracting the double of the three joint Lives, *viz.* 14.82, the Remainder is 14.24. Hence ſuppoſing $T=$ 14.24, then $\overline{1-dT}\times f$ will be found to be 1076 *l.* and this is the Value of A's Expectation.

Laſtly, If A was not to have the Annuity before the Extinction of the three Lives, ſuppoſe the Value of the three Lives $=V$, then the Expectation of A would be worth $\overline{1-dV}\times f$, which in this Caſe is 465 *l.*

PRO-

PROBLEM XXVII.

To determine the Fines to be paid for renewing any Number of Years in a College-Leaſe of twenty, and alſo what Rate of Intereſt is made by a Purchaſer, who may happen to give an advanced Price for the ſame, upon Supposition that the Contractor is allowed 8 per Cent. *of his Money.*

ALTHO' the Problem here propoſed does not ſeem to relate to the Subject of this Book, yet as ſome uſeful Concluſions may be derived from the Solution of it, I have thought fit to inſert it in this place.

Table of Fines.

1	0.2146	8	2.2821	15	5.8254
2	0.4463	9	2.6792	16	6.5060
3	0.6965	10	3.1081	17	7.2411
4	0.9666	11	3.5713	18	8.0349
5	1.2587	12	4.0715	19	8.9922
6	1.4133	13	4.6118	20	9.8181
7	1.9144	14	5.1953		

If

IF a Purchaſer gives the Original Contractor 11 Years Purchaſe for his Leaſe of 20, he makes above $6\frac{1}{2}$ *per Cent.* of his Money.

IF he gives 12 Years Purchaſe for the ſame, he makes above 5 *l.* 8 *s. per Cent.* of his Money.

IF he gives 13 Years Purchaſe, he makes $4\frac{1}{2}$ *per Cent.* of his Money.

PROBLEM XXVIII.

To determine the Fines to be paid for renewing any Number of Years in a College-Leaſe of One and Twenty, as alſo what Rate of Intereſt is made by a Purchaſer who may happen to give an advanced Price for the ſame, upon Suppoſition that the Contractor is allowed 8 per Cent. *of his Money.*

Table

Table of Fines.

1	0.1987	8	2.1131	15	5.3940
2	0.4133	9	2.2808	16	6.0241
3	0.6450	10	2.8779	17	6.7047
4	0.8952	11	3.3068	18	7.4398
5	1.1653	12	3.7700	19	8.2336
6	1.4574	13	4.2702	20	9.0909
7	1.6120	14	4.8105	21	10.0168

HE that gives 11 Years Purchaſe, inſtead of 10.0168 for renewing his Leaſe for 21 Years, makes 6 *l.* 15 *s. per Cent.* of his Money.

HE who gives 12 Years Purchaſe for the ſame, makes very near 5 *l.* 16 *s. per Cent.* of his Money.

HE who gives 13 Year Purchaſe for the ſame, makes a little more than 4 *l.* 16 *s. per Cent.* of his Money.

PRO-

The Values of Annuities for Lives having been calculated in this Book, upon a Suppoſition that the Payments are made Yearly, and there being ſome Occaſions wherein it is ſtipulated that the Payments ſhould be made Half-Yearly, I have thought fit to add the two following Problems, whereby, 1°. *It is ſhewn what the Half-Yearly Payments ought to be, if the Price of the Purchaſe is preſerved.* 2°. *How the Price of the Purchaſe ought to be increaſed, if the Half-Yearly Payments are required to be the Half of the Yearly Payments.*

PROBLEM XXIX.

An Annuity being given, to find what Half-Yearly Payments will be equivalent to it, when Intereſt of Money is 4, 5, *or* 6 per Cent.

SOLUTION.

TAKE half of the Annuity, and from that half ſubtract its 100th, or 80th, or 68th Part, according as the

Intereſt is 4, 5, or 6 *per Cent.* and the Remainder will be the Value of the Half-Yearly Payments required; thus, if the Annuity was 100 *l.* the Half-Yearly Payments would reſpectively be 49 *l.* 10 *s.* 49 *l.* 7 *s.* 6 *d.* 49 *l.* 5 *s.* 3 *d.* nearly.

PROBLEM XXX.

The preſent Value of an Annuity being given, to find how much this preſent Value ought to be increaſed, when it is required that the Payments ſhall be Half-Yearly, and alſo one half of the Yearly Payments, when Intereſt is at 4, 5, *or* 6 per Cent.

SOLUTION.

To the preſent Value of the Annuity, add reſpectively its 99th, 79th, or 67th, and the Sums will be the Values increaſed.

As there are ſome Perſons who may be deſirous to ſee a general Solution of the

the two laſt Problems, I have thought fit to add what follows.

IN the firſt of the two laſt Problems, let A be the Yearly Payments agreed on, and B the Half-Yearly Payments required, r the Yearly Rate of Intereſt, then $B = \frac{r^{\frac{1}{2}} - 1}{r - 1} \times A$. In the ſecond, let M be the preſent Value of the Yearly Payments, P the preſent Value of thoſe that are to be Half-Yearly, then $P = \frac{\frac{1}{2} \times \overline{r - 1}}{r^{\frac{1}{2}} - 1} \times M$.

TABLE I.

The present Value of an Annuity of one Pound for any Number of Years not exceeding 100, *Interest at four* per Cent.

Years.	Values.	Years.	Values.	Years.	Values.	Years.	Values.
1	0.9615	26	15.9827	51	21.6714	76	23.7311
2	1.8860	27	16.3295	52	21.7475	77	23.7799
3	2.7750	28	16.6630	53	21.8726	78	23.8268
4	3.6298	29	16.9837	54	21.9929	79	23.8720
5	4.4518	30	17.2920	55	22.1086	80	23.9153
6	5.2421	31	17.5884	56	22.2198	81	23.9571
7	6.0020	32	17.8735	57	22.3267	82	23.9972
8	6.7327	33	18.1476	58	22.4295	83	24.0357
9	7.4353	34	18.4111	59	22.5284	84	24.0728
10	8.1108	35	18.6646	60	22.6234	85	24.1085
11	8.7604	36	18.9082	61	22.7148	86	24.1428
12	9.3850	37	19.1425	62	22.8027	87	24.1757
13	9.9856	38	19.3678	63	22.8872	88	24.2074
14	10.5631	39	19.5844	64	22.9685	89	24.2379
15	11.1183	40	19.7927	65	23.0466	90	24.2672
16	11.6522	41	19.9930	66	23.1218	91	24.2954
17	12.1656	42	20.1856	67	23.1940	92	24.3225
18	12.6592	43	20.3707	68	23.2635	93	24.3486
19	13.1339	44	20.5488	69	23.3302	94	24.3736
20	13.5903	45	20.7200	70	23.3945	95	24.3977
21	14.0291	46	20.8846	71	23.4562	96	24.4209
22	14.4511	47	21.0429	72	23.5156	97	24.4431
23	14.8568	48	21.1951	73	23.5727	98	24.4646
24	15.2469	49	21.3414	74	23.6276	99	24.4851
25	15.6220	50	21.4821	75	23.6804	100	24.5049

TABLE II.

The preſent Value of an Annuity of one Pound, to continue ſo long as a Life of a given Age is in being, Intereſt being eſtimated at 4 per Cent.

Age	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	13.36	26	15.19	51	11.13	76	3.91
2	14.54	27	15.04	52	10.92	77	3.52
3	15.43	28	14.94	53	10.70	78	3.11
4	15.89	29	14.81	54	10.47	79	2.70
5	16.21	30	14.68	55	10.24	80	2.28
6	16.50	31	14.54	56	10.01	81	1.85
7	16.64	32	14.41	57	9.77	82	1.40
8	16.79	33	14.27	58	9.52	83	0.95
9	16.88	34	14.12	59	9.27	84	0.48
10	16.88	35	13.98	60	9.01	85	0.00
11	16.79	36	13.82	61	8.75	86	0.00
12	16.64	37	13.67	62	8.48		
13	16.60	38	13.52	63	8.20		
14	16.50	39	13.36	64	7.92		
15	16.41	40	13.20	65	7.63		
16	16.31	41	13.02	66	7.33		
17	16.21	42	12.85	67	7.02		
18	16.10	43	12.68	68	6.75		
19	15.99	44	12.50	69	6.39		
20	15.89	45	12.32	70	6.06		
21	15.78	46	12.13	71	5.72		
22	15.67	47	11.94	72	5.38		
23	15.55	48	11.74	73	5.02		
24	15.43	49	11.54	74	4.66		
25	15.31	50	11.34	75	4.29		

TABLE III.

The present Value of an Annuity of one Pound for any Number of Years, not exceeding 100, *Interest at five* per Cent.

Years.	Values.	Years.	Values.	Years.	Values.	Years.	Values.
1	0.9523	26	14.3751	51	18.3389	76	19.5094
2	1.8594	27	14.6430	52	18.4180	77	19.5328
3	2.7232	28	14.8981	53	18.4934	78	19.5550
4	3.5459	29	15.1410	54	18.5651	79	19.5762
5	4.3294	30	15.3724	55	18.6334	80	19.5964
6	5.0756	31	15.5928	56	18.6985	81	19.6156
7	5.7863	32	15.8026	57	18.7605	82	19.6339
8	6.4632	33	16.0025	58	18.8195	83	19.6514
9	7.1078	34	16.1929	59	18.8757	84	19.6680
10	7.7212	35	16.3741	60	18.9292	85	19.6838
11	8.3064	36	16.5468	61	18.9802	86	19.6988
12	8.8632	37	16.7112	62	19.0288	87	19.7132
13	9.3935	38	16.8678	63	19.0750	88	19.7268
14	9.8986	39	17.0170	64	19.1191	89	19.7398
15	10.3796	40	17.1590	65	19.1610	90	19.7522
16	10.8377	41	17.2943	66	19.2010	91	19.7640
17	11.2740	42	17.4232	67	19.2390	92	19.7752
18	11.6895	43	17.5459	68	19.2753	93	19.7859
19	12.0853	44	17.6627	69	19.3098	94	19.7961
20	12.4622	45	17.7740	70	19.3426	95	19.8058
21	12.8211	46	17.8800	71	19.3739	96	19.8151
22	13.1630	47	17.9810	72	19.4037	97	19.8239
23	13.4885	48	18.0771	73	19.4321	98	19.8323
24	13.7986	49	18.1687	74	19.4592	99	19.8403
25	14.0939	50	18.2559	75	19.4849	100	19.8479

TABLE IV.

The present Value of an Annuity of one Pound, to continue so long as a Life of a given Age is in being, Interest at five per Cent.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	11.96	26	13.37	51	10.17	76	3.78
2	12.88	27	13.28	52	9.99	77	3.41
3	13.55	28	13.18	53	9.82	78	3.03
4	13.89	29	13.09	54	9.63	79	2.64
5	14.12	30	12.99	55	9.44	80	2.23
6	14.34	31	12.88	56	9.24	81	1.81
7	14.47	32	12.78	57	9.04	82	1.38
8	14.53	33	12.67	58	8.83	83	0.94
9	14.60	34	12.56	59	8.61	84	0.47
10	14.60	35	12.45	60	8.39	85	0.00
11	14.53	36	12.33	61	8.16	86	0.00
12	14.47	37	12.21	62	7.93		
13	14.41	38	12.09	63	7.68		
14	14.34	39	11.96	64	7.43		
15	14.27	40	11.83	65	7.18		
16	14.20	41	11.70	66	6.91		
17	14.12	42	11.57	67	6.64		
18	14.05	43	11.43	68	6.36		
19	13 97	44	11.29	69	6.97		
20	13.89	45	11.14	70	5.77		
21	13.81	46	10.99	71	5.47		
22	13.72	47	10.84	72	5.15		
23	13.64	48	10.68	73	4.82		
24	13.55	49	10.51	74	4.49		
25	13.46	50	10.35	75	4.14		

TABLE V.

The present Value of an Annuity of one Pound for any Number of Years not exceeding 100, *Interest at* 6 per Cent.

Years.	Values.	Years.	Values.	Years.	Values.	Years.	Values.
1	0.9433	28	13.0031	51	15.8130	76	16.4677
2	1.8333	27	13.2105	52	15.8613	77	16.4790
3	2.6730	28	13.4061	53	15.9069	78	16.4896
4	3.4651	29	13.5907	54	15.9499	79	16.4996
5	4.2123	30	13.7648	55	15.9905	80	16.5091
6	4.9173	31	13.9290	56	16.0288	81	16.5180
7	5.5823	32	14.0840	57	16.0649	82	16.5264
8	6.2097	33	14.2302	58	16.0989	83	16.5343
9	6.8016	34	14.3681	59	16.1311	84	16.5418
10	7.3600	35	14.4982	60	16.1614	85	16.5489
11	7.8868	36	14.6209	61	16.1900	86	16.5556
12	8.3838	37	14.7367	62	16.2170	87	16.5618
13	8.8526	38	14.8460	63	16.2424	88	16.5678
14	9.2949	39	14.9490	64	16.2664	89	16.5734
15	9.7122	40	15.0462	65	16.2891	90	16.5786
16	10.1058	41	15.1380	66	16.3104	91	16.5836
17	10.4772	42	15.2245	67	16.3306	92	16.5883
18	10.8276	43	15.3061	68	16.3496	93	16.5928
19	11.1581	44	15.3831	69	16.3676	94	16.5969
20	11.4699	45	15.4558	70	16.3845	95	16.6009
21	11.7640	46	15.5243	71	16.4005	96	16.6046
22	12.0415	47	15.5890	72	16.4155	97	16.6081
23	12.3033	48	15.6500	73	16.4297	98	16.6114
24	12.5503	49	15.7075	74	16.4431	99	16.6145
25	12.7833	50	15.7618	75	16.4558	100	16.6175

TABLE VI.

The present Value of an Annuity of one Pound, to continue so long as a Life of a given Age is in being, Interest being estimated at 6 per Cent.

Age	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	10.80	26	11.90	51	9.34	76	3.66
2	11.53	27	11.83	52	9.20	77	3.31
3	12.04	28	11.76	53	9.04	78	2.95
4	12.30	29	11.68	54	8.90	79	2.57
5	12.47	30	11.61	55	8.72	80	2.18
6	12.63	31	11.53	56	8.56	81	1.78
7	12.74	32	11.45	57	8.38	82	1.36
8	12.79	33	11.36	58	8.20	83	0.92
9	12.84	34	11.60	59	8.02	84	0.77
10	12.84	35	11.18	60	7.83	85	0.00
11	12.79	36	11.09	61	7.63	86	0.00
12	12.74	37	11.00	62	7.42		
13	12.69	38	10.90	63	7.21		
14	12.63	39	10.80	64	7.00		
15	12.58	40	10.70	65	6.77		
16	12.53	41	10.60	66	6.53		
17	12.47	42	10.50	67	6.22		
18	12.41	43	10.37	68	6.03		
19	12.36	44	10.26	69	5.77		
20	12.30	45	10.14	70	5.50		
21	12.23	46	10.02	71	5.22		
22	12.17	47	9.90	72	4.93		
23	12.11	48	9.76	73	4.63		
24	12.04	49	9.63	74	4.32		
25	11.97	50	9.49	75	4.00		

APPENDIX:

Containing the Demonſtrations of ſome of the principal Propoſitions in the foregoing Treatiſe.

CHAPTER I.

I Obſerved formerly, that upon Suppoſition that the Decrements of Life were in Arithmetic Progreſſion, the Concluſions derived from thence, would very little vary, from thoſe that could be deduced from the Table of Obſervations made at *Breſlaw*, concerning the Mortality of Mankind; which Table was about fifty Years ago inſerted by Dr. *Halley* in the *Philoſophical Tranſactions*, together with ſome Calculations concerning the Values of Lives according to a given Age.

UPON the foregoing Principle, I ſuppoſed that if n repreſented the Com-

plement of Life, the Probabilities of living 1, 2, 3, 4, 5, &c. Years, would be expreſſed by the following Series, $\frac{n-1}{n}$, $\frac{n-2}{n}$, $\frac{n-3}{n}$, $\frac{n-4}{n}$, $\frac{n-5}{n}$, &c. and conſequently that the Value of a Life, whoſe complement is n, would be expreſſed by the Series $\frac{n-1}{nr}+\frac{n-2}{nrr}+\frac{n-3}{nr^3}+\frac{n-4}{nr^4}+\frac{n-5}{nr^5}$, &c. the Sum of which I have aſſerted in my third Page to be $\frac{1-\frac{r}{n}P}{r-1}$, where the Signification of the Quantities P and r is explained.

As the Reaſonings that led me to that general Expreſſion, require ſomething more than an ordinary Skill in the Doctrine of Series, I ſhall forbear to mention them in this Place, and content my ſelf with pointing out to the Reader a Method, whereby he may ſatisfy himſelf of the Truth of that Theorem, provided he underſtand ſo much of a Series, as to be able to ſum up a Geometric Progreſſion.

DE-

DEMONSTRATION.

$$P = \frac{1}{r} + \frac{1}{rr} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} \cdots \cdots \frac{1}{r^n}.$$

Therefore,

$$rP = 1 + \frac{1}{r} + \frac{1}{rr} + \frac{1}{r^3} + \frac{1}{r^4} \cdots + \frac{1}{r^{n-1}}.$$

And

$$\frac{rP}{n} = \frac{1}{n} + \frac{1}{nr} + \frac{1}{nrr} + \frac{1}{nr^3} + \frac{1}{nr^4} \cdots \cdots + \frac{1}{nr^{n-1}}.$$

Therefore,

$$1 - \frac{rP}{n} = \frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nrr} - \frac{1}{nr^3} - \frac{1}{nr^4} \cdots \frac{1}{nr^{n-1}}.$$

But this is to be divided by $r-1$, or multiplied by

$$\frac{1}{r-1} = \frac{1}{r} + \frac{1}{rr} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6}, \text{ \&c.}$$

Then multiplying actually thoſe two Series's together, the Product will be found to be

$$\begin{aligned}
&\frac{n-1}{nr} - \frac{1}{nrr} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} \text{ \&c.}\\
&\quad + \frac{n-1}{nrr} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} \text{ \&c.}\\
&\qquad + \frac{n-1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} \text{ \&c.}\\
&\qquad\quad + \frac{n-1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} \text{ \&c.}\\
&\qquad\qquad + \frac{n-1}{nr^5} - \frac{1}{rn^6} \text{ \&c.}\\
&\qquad\qquad\quad + \frac{n-1}{nr^6} \text{ \&c.}
\end{aligned}$$

And

And adding the Terms of the perpendicular Columns together, we ſhall have $\frac{n-1}{nr} + \frac{n-2}{nr} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6}$ &c. which conſequently is equal to $\frac{1-\frac{r}{n}P}{r-1}$, which was to be demonſtrated.

IF it be required that upon the Failing of a Life, ſuch Part of the Annuity ſhould be paid, as may be proportional to the time elapſed from the beginning of the laſt Year, to the time of the Life's failing, then the Value of the Life will be $\frac{1}{r-1} - \frac{1}{an}P$, wherein a repreſents the hyperbolic Logarithm of the Rate of Intereſt.

BUT becauſe there are no Tables printed of hyperbolic Logarithms, and that the Reduction of a common Logarithm to an hyperbolic is ſomewhat laborious, it will be ſufficient here to ſet down the hyperbolic Logarithms of 1.04, 1.05, 1.06, which are reſpectively, 0.03922, 0.04879, 0.05825, or $\frac{2}{51}$, $\frac{2}{41}$, $\frac{6}{103}$ nearly.

CHAPTER

CHAPTER II.

Explaining the Rules of combined Lives.

SUPPOSING a fictitious Life, whoſe Number of Chances to continue in being, from Year to Year, be conſtantly equal to a, and the Number of Chances for failing be conſtantly equal to b, ſo that the odds of its continuing during the Space of any one Year, be to its failing in the ſame interval of Time conſtantly as a to b, the Value of an Annuity upon ſuch Life would be eaſily found.

FOR, if we make $a+b=s$, the Probabilities of living 1, 2, 3, 4, 5, &c. Years would be repreſented by the Series $\frac{a}{s}$, $\frac{aa}{ss}$, $\frac{a^3}{s^3}$, $\frac{a^4}{s^4}$, $\frac{a^5}{s^5}$, &c. continued to eternity, and conſequently the Value of an Annuity upon ſuch Life would be expreſſed by this new Series $\frac{a}{sr}+\frac{aa}{ssrr}+\frac{a^3}{s^3r^3}+\frac{a^4}{s^4r^4}$ &c. which being a geometric Progreſſion perpetually decreaſing, the Sum of it will be found

found to be $\frac{a}{sr-a}$; thus, if a ſtands for 21, and b for 1, and alſo r for 1.05, the Value of ſuch Life would be 10 Years Purchaſe.

FROM theſe Premiſes the following Corollaries may be drawn:

COROLLARY I.

AN Annuity upon a fictitious Life being given, the Probability of its continuing one Year in being is alſo given; for let the Value be $=M$, then $\frac{a}{f}=\frac{Mr}{M+1}$.

COROLLARY II.

IF a Life, whoſe Value is deduced from our Tables, is found to be worth 10 Years purchaſe, then ſuch Life is equivalent to a fictitious Life, whoſe Number of Chances for continuing one Year, is to the Number of Chances for its failing in that Year, as 21 to 1.

COROLLARY III.

WHEREFORE having taken the Value of a Life from our Tables, or calculated

culated it according to the Rules preſcribed, we may transfer the Value of that Life to that of a fictitious Life, and find the Number of Chances it would have for continuing or failing Yearly.

COROLLARY IV.

AND the Combination of two or more *real Lives* will be very near the ſame as the Combination of ſo many correſponding *fictitious Lives*; and therefore an Annuity granted upon one or more real Lives, is nearly of the ſame Value as an Annuity upon a fictitious Life.

THESE things being premiſed, it will not be difficult to determine the Value of an Annuity upon two or three, or as many joint Lives as may be aſſigned.

FOR let x repreſent the Probability of one Life's continuing from Year to Year, and y the Probability of another Life's continuing the ſame Time, then

according to the Principles of the Doctrine of Chances, the Terms

xy, $xxyy$, x^3y^3, x^4y^4, x^5y^5, &c.

will reſpectively repreſent the Probabilities of continuing together, 1, 2, 3, 4, 5, &c. Years; then the Value of an Annuity upon the two joint Lives, will be $\frac{xy}{r}+\frac{xxyy}{rr}+\frac{x^3y^3}{r^3}+\frac{x^4y^4}{r^4}+\frac{x^5y^5}{r^5}$ &c. which being a Geometrical Progreſſion perpetually decreaſing, the Sum of it will be found to be $\frac{xy}{r-xy}$; let now M be put for the Value of the firſt Life, and P for the Value of the ſecond, then by our firſt Corollary it appears that $x=\frac{Mr}{M+1}$, and $y=\frac{Pr}{P+1}$, and therefore having written theſe Values of x and y in the Expreſſion $\frac{xy}{r-x}$, which is the Value of the two joint Lives, it will be changed into $\frac{MPr}{\overline{M+1}\times\overline{P+1}-MPr}$, which is the ſame Theorem that I had given in my firſt Edition, which came out ſeventeen Years ago.

IT

IT is true that in the ſixth Page of this Book I have given a Theorem which ſeems very different from this, making the Value of the joint Lives to be $\frac{MP}{M+P-dMP}$, wherein d repreſents the Intereſt of 1 *l.* and yet I may aſſure the Reader, that this laſt Expreſſion is originally derived from the firſt, and that whether one or the other is uſed, the Concluſions will very little differ, but the firſt Theorem is better adapted to Annuities paid in Money, it being cuſtomary that the laſt Payment, whether it be Yearly or Half-Yearly, is loſt to the Purchaſer; whereas the ſecond Theorem is better fitted to Annuities paid by a Grant of Lands, whereby the Purchaſer makes Intereſt of his Money to the laſt Moment of his Life, for which Reaſon I have choſe to uſe the laſt Expreſſion in my Book.

BY following the ſame Method of Inveſtigation, we ſhall find that if M, P, Q, denote three ſingle Lives, an

Annuity upon thoſe joint Lives will be $\frac{MP\mathfrak{Q}rr}{\overline{M+1\times P+1\times \mathfrak{Q}+1}-MP\mathfrak{Q}rr}$; in the Caſe of Annuities payable in Money, or $\frac{MP\mathfrak{Q}}{MP+M\mathfrak{Q}+P\mathfrak{Q}-2dMP}$, in the Caſe of Annuities paid by a Grant of Lands.

CHAPTER III.

Containing the Demonſtration of the Rules given in Problems 4th and 5th, for determining the longeſt Life.

LET x and y repreſent the reſpective Probabilities which two Lives have of continuing one Year in being, therefore $1-x$ is the Probability of the firſt Life's failing in one Year, and $1-y$ the Probability of the ſecond Life's failing in one Year: Therefore multiplying theſe two Probabilities together, the Product $1-x-y+xy$ will repreſent the Probability of the two Lives failing in one Year, and if this be ſubtracted from Unity, the Remainder

der $x+y-xy$ will expreſs the Probability of one at leaſt of the two Lives outliving one Year, which is ſufficient for eſtabliſhing the firſt Year's Rent.

AND for the ſame Reaſon $xx+yy-xxyy$ will expreſs the Probability of one at leaſt of the two Lives outliving two Years, which is ſufficient to eſtabliſh the ſecond Year's Rent.

FROM the two Steps we have taken, it plainly appears that the longeſt of two Lives is expreſſible by the three following Series;

$$\frac{x}{r}+\frac{xx}{rr}+\frac{x^3}{r^3}+\frac{x^4}{r^4}+\frac{x^5}{r^5}$$
$$+\frac{y}{r}+\frac{yy}{rr}+\frac{y^3}{r^3}+\frac{y^4}{r^4}+\frac{y^5}{r^5}$$
$$-\frac{xy}{r}-\frac{xxyy}{rr}-\frac{x^3y^3}{r^3}-\frac{x^4y^4}{r^4}-\frac{x^5y^5}{r^5}$$

Whereof the firſt repreſents an Annuity upon the firſt Life, the ſecond an Annuity upon the ſecond Life, and the third an Annuity upon the two joint Lives; and therefore we may conclude that an Annuity upon the longeſt of two Lives, is the Difference between the

the Sum of the Values of the ſingle Lives, and the Value of the joint Lives, which has been expreſſed in my 14th Page by the Symbols $M+P-\overline{MP}$.

IN the ſame manner it will be found that if x, y, z, repreſent the reſpective Probabilities of three Lives living one Year, then the Probability of their not failing all three in one Year will be expreſſed by $x+y+z-xy-xz-yz+xyz$, which is ſufficient to ground this Concluſion, that an Annuity upon the longeſt of three Lives, is the Sum of the ſingle Lives, *minus* the Sum of the joint Lives, *plus* the three joint Lives, which has been expreſſed by me, by the Symbols $M+P+Q-\overline{MP}-\overline{MQ}-\overline{PQ}+\overline{MPQ}$.

FROM the foregoing Concluſions, it is eaſily perceived how the Value of the longeſt of any Number of Lives ought to be determined, *viz.* by the Sum of the Values of the ſingle Lives, *minus* the Sum of the Values of all the joint Lives taken two and two

two, *plus* the Sum of all the joint Lives taken three and three, *minus* the Sum of all the joint Lives taken four and four, and ſo on by alternate Additions and Subtractions.

CHAPTER IV.

Containing the Demonſtrations of what has been ſaid concerning Reverſions, and the Value of one Life after one or more Lives.

1°. It plainly appears that the preſent Value of a Reverſion after one Life, is the Difference between the Perpetuity, and the Value of the Life in Poſſeſſion: Thus, if the Life in Poſſeſſion be worth 14 Years purchaſe, and that I have the Reverſion after that Life, and have a mind to ſell it, I muſt have for it 11 Years purchaſe, which is the Difference between the Perpetuity 25, and 14 the Value of the Life, when Money is rated at 4 *per Cent.*

2°. IT

2°. IT is as evident that the Reverſion after two, three, or more Lives, is the Difference between the Perpetuity, and the longeſt of all the Lives.

BUT the Value of a Life after one or more Lives not being ſo obvious, I think it is proper to inſiſt upon it more largely; let x therefore repreſent the Probability of the Expectant's Life continuing one Year in being, and y the Probability of the ſecond Life's continuing alſo one Year in being, and therefore $1-y$ is the Probability of that ſecond Life's failing in that Year; from which it follows, according to the Doctrine of Chances, that the Probability of the firſt Life's continuing one Year, and of the ſecond's failing in that Year, is $x \times \overline{1-y}$, or $x-xy$, which is a ſufficient foundation for drawing the following Concluſion, *viz.* that the Value of the firſt Life after the ſecond is the Value of that firſt Life *minus* the Value of the two joint Lives, which I have expreſſed by the Symbols $M-\overline{MP}$.

IN

IN the ſame manner, if x, y, z, repreſent the reſpective Probabilities of three Lives continuing one Year, then $x \times \overline{1-y} \times \overline{1-z}$, will repreſent the Probability of the firſt Life's continuing one Year, and of the other two Lives failing in that Year; but the foregoing Expreſſion is brought, by actual Multiplication, to its Equivalent $x-xy-xz+xyz$, from whence can be deduced by meer Inſpection the Rule given in our 20th *pag. viz.* that the preſent Value of the firſt Life's Expectation after the Failing of the other two, is $M-\overline{MP}-\overline{MQ}+\overline{MPQ}$.

CHAPTER V.

Containing the Demonſtration of what has been aſſerted in the Solution of the 10th and 26th Problems.

It appears from the Solution of the 10th Problem, that M''' denoting the preſent Value of an Annuity to continue ſo long as three Lives of the ſame

Age ſubſiſt together, let us ſuppoſe that n denotes the Number of Years during which the Annuity will continue; then ſuppoſing r to expreſs the Rate of Intereſt, it is well known that the preſent Value of that Annuity will be $\frac{1-\frac{1}{r^n}}{r-1}$, wherefore we have the Equation $M''' = \frac{1-\frac{1}{r^n}}{r-1}$, or making $r-1=d$, $M''' = \frac{1-\frac{1}{r^n}}{d}$, from whence will be deduced $\frac{1}{r^n} = 1-dM'''$, and conſequently $r^n = \frac{1}{1-dM'''}$. Now let us ſuppoſe that a Sum f is to be received to eternity at the equal Intervals of Time, denoted by n, and that we want to find the preſent Value of it; it is plain to thoſe who have made ſome Proficiency in Algebra, that $\frac{f}{r^n-1}$ is the preſent Value of it, let us therefore in the room of r^n ſubſtitute its Value found before, *viz.* $\frac{1}{1-dM'''}$

$\frac{1}{1-dM^{m}}$, and then $r^{n}-1$ will be found equal to $\frac{dM^{m}}{1-dM^{m}}$, and conſequently $\frac{f}{r^{n}-1}=\frac{1-dM^{m}}{dM^{m}}\times f$, which is the Aſſertion of my 23d *Page*.

Now it will be eaſy to find n, for let us ſuppoſe $\frac{1}{1-dM^{m}}=T$, then $r^{n}=T$, and therefore $n=\frac{log.\ T}{log.\ r}$.

THE 25th Problem has ſome Affinity with the 10th, in the former it was required to know the preſent Value of a Sum f, payable at the Failing of any one of three equal Lives, but in the latter the three Lives are ſuppoſed unequal; but beſides, it is extended to two other Caſes, *viz.* to the preſent Value of a Sum f to be paid after the Failing of any two of the Lives, as alſo to the preſent Value of a Sum f to be paid after the Failing of the three Lives.

FOR in the firſt Caſe, let us imagine an Annuity to be paid as long as the three Lives are in being; or, which is the ſame thing, till one of the Lives fails,

fails, and let us ſuppoſe that R repreſents the Value of the three joint Lives; let us alſo ſuppoſe that n is the Number of Years after which this will happen, and that d is the Intereſt of 1*l.* therefore $\frac{f}{r^n}$ is the preſent Value of the Sum f to be then paid, but $R = \frac{1-\frac{1}{r^n}}{d}$, therefore $\frac{1}{r^n} = 1 - dR$, and therefore $\frac{f}{r^n} = \overline{1-dR} \times f$.

BUT the ſecond Caſe has ſomething more of Difficulty, and therefore I ſhall enlarge a little more upon it; let us imagine now that there is an Annuity to continue not only as long as the three equal Lives are in being, but as long as any two of the ſaid Lives are in being; now in order to find the preſent Value of the ſaid Annuity, let us ſuppoſe that x, y, z, repreſent the reſpective Probabilities of the ſaid Lives continuing one Year. Therefore,

1°. xyz repreſents the Probability of their all outliving the Year.

2°.

2°. $xy \times 1-z$, or $xy-xyz$ repreſents the Probability of the two firſt outliving the Year, and of the third failing in that Year.

3°. $xz \times \overline{1-y}$ repreſents the Probability of the firſt and third's outliving the Year, and of the ſecond's failing in that Year.

4°. $yz \times \overline{1-x}$, or $yz-xyz$ repreſents the Probability of the ſecond and third's outliving the Year, and of the firſt's failing in that Year.

THEN adding thoſe ſeveral Products together, their Sum will be found equal to $xy+xz+yz-2xyz$, which is an Indication that the preſent Value of an Annuity to continue as long as two of the ſaid Lives are in being is $\overline{MP}+\overline{MQ}+\overline{PQ}-\overline{2MPQ}$, which we may ſuppoſe $=T$.

LET us now compare this with an Annuity certain to continue n Years, the Rate of Intereſt being ſuppoſed $=r$, and $r-1=d$, then we ſhall have

have the Equation $\frac{1-\frac{1}{r^n}}{d} = T$, from whence we ſhall find $\frac{1}{r^n} = 1-dT$, and conſequently $\frac{f}{r^n}$ which is the preſent Value of the Expectation required, is $= \overline{1-dT} \times f$.

By the ſame Method of Proceſs, we may find the preſent Value of an Annuity to continue ſo long as any one of the three Lives in queſtion is ſubſiſting, for let x, y, z, repreſent the ſame things as before.

1°. xyz repreſents the Probability of the three Lives outliving the firſt Year.

2°. $xy + xz + yz - 3xyz$ repreſents the Probability of two of them outliving the Year, and of the third's failing in that Year.

3°. $x \times \overline{1-y} \times \overline{1-z}$, or $x - xy - xz + xyz$ repreſents the Probability of the firſt Life's outliving the Year, and of the other two failing in that Year.

4°.

4°. $y \times \overline{1-x} \times \overline{1-z}$, or $y-xy-zy+xyz$ repreſents the Probability of the ſecond Life's outliving the Year, and of the other two failing in that Year.

5°. $z \times \overline{1-x} \times \overline{1-y}$, or $z-xz-yz+xyz$ repreſents the Probability of the third Life's outliving the Year, and of the other two failing in that Year.

Now the Sum of all this is $x+y+z-xy-xz-yz+xyz$, which is an Indication that the Value of an Annuity to continue as long as any one of three Lives is in being ought to be expreſſed by $M+P+Q-\overline{MP}-\overline{MQ}-\overline{PQ}+\overline{MPQ}$, and this laſt Caſe may be look'd upon as a Confirmation of the Rule given in our 5th Problem.

CHAPTER VI.

Containing the Demonſtration of what has been ſaid concerning ſucceſſive Lives pag. 33. *and following.*

WHAT has been there ſaid amounts to this; The preſent Values of Annuities certain for any particular Number of Years being given, to find the preſent Value of an Annuity to continue as long as the Sum of thoſe Years.

LET us ſuppoſe that M repreſents the preſent Value of an Annuity to continue n Years, and that P repreſents the preſent Value of an Annuity to continue p Years; the firſt Queſtion is, how from theſe *Datas* to find the preſent Value of an Annuity to continue $n+p$ Years, the Inveſtigation of which is as follows; let r be the Rate of Intereſt, and ſuppoſe $r-1$ which denotes the Intereſt of 1 $l.=d$, then,

1°. $M=\frac{1-\frac{1}{r^n}}{d}$, therefore $\frac{1}{r^n}=1-dM$, and

and for the ſame Reaſon $\frac{1}{r^p} = 1 - dP$. Therefore $\frac{1}{r^{n+p}} = \overline{1-dM} \times \overline{1-dP} = 1 - dM - dP + ddMP$. Let now S be ſuppoſed to be the Value of the Annuity which is to continue $n+p$ Years, then $\frac{1}{r^{n+p}} = 1 - df$. Therefore $1 - df = 1 - dM - dP + ddMP$; then ſubtracting Unity on both Sides, dividing all by d, and changing the Signs, we ſhall have $f = M + P - dMP$.

2°. By the ſame Method of Proceſs, it will be eaſy to find that if M, P, Q, repreſent Annuities to continue for the reſpective Number of Years n, p, q, then the Value of an Annuity to continue $n+p+q$ Years will be $M+P+Q-dM-dP-dQ+ddMPQ$, the Continuation of which is obvious.

Let us now ſuppoſe that the Intervals n, p, q, are equal, then the Values M, P, Q are alſo equal, in which caſe, the foregoing Canon will be changed into this, $3M - 3dMM + d^3M^3$, or $\frac{3dM - 3ddMM + d^3M}{d}$,

but if this Numerator be ſubtracted from Unity, the remainder will be $1-3dM+3ddMM-d^3M^3=\overline{1-dM}^3$, and ſubtracting this again from Unity, the original Numerator will be reſtored, and will be equivalent to $1-\overline{1-dM}^3$, and conſequently, if M repreſents the Value of an Annuity to continue a certain Number of Years, then $\frac{1-\overline{1-dM}^3}{d}$ will repreſent the Value of an Annuity to continue three times as long.

AND univerſally, if M ſtands for the Value of an Annuity to continue a certain Number of Years, then $\frac{1-\overline{1-dM}^n}{d}$ will repreſent the Value of an Annuity to continue n times as long.

AND if n were infinite, I ſay that $\overline{1-dM}^n$ would be $=0$, from whence the Value would be $=\frac{1}{d}$ or $\frac{1}{r-1}$, which repreſents the Value of the Perpetuity.

BUT

BUT that there may remain no ſcruple about what we have aſſerted above, that in the Caſe of n being infinite, $\overline{1-dM}^{n}$ would vaniſh; I prove it thus, $\frac{1}{d} > M$, therefore $1 > dM$, therefore $1-dM$ is a Fraction leſs than Unity: now it is well known that a Fraction leſs than Unity being raiſed to an infinite Power, is nothing, and was therefore ſafely neglected.

CHAPTER VII.

Containing the Demonſtration of what has been aſſerted in the Pages 73 *and* 74, *concerning half-yearly Payments, as alſo the Inveſtigation of ſome Theorems relating to that Subject.*

IT is well known that if an Annuity A is to continue n Years, the preſent Value of it is $\frac{A-\frac{A}{r^n}}{r-1}$; ſuppoſing r to repreſent the Rate of Intereſt;

tereſt; now to make a proper Application of this Theorem to half-Yearly Payments, I look upon n as repreſenting indifferently the Number of Payments and the Number of Years; let us now ſuppoſe a half-Yearly Rent B of the ſame preſent Value as the former, and to continue as long, then the Number of Payments in this caſe will be $2n$, but the Rate of Intereſt, inſtead of being r, is now $r^{\frac{1}{2}}$, which being raiſed to the Power $2n$, will be r^n as before; for which Reaſon the preſent Value of the half-Yearly Payments is $\frac{B-\frac{B}{r^n}}{r^{\frac{1}{2}}-1}$, but by Hypotheſis, the preſent Values of the Yearly and half-yearly Payments are the ſame; therefore $\frac{A-\frac{A}{r^n}}{r-1} = \frac{B-\frac{B}{r^n}}{r^{\frac{1}{2}}-1}$, and dividing both ſides of the Equation by $1-\frac{1}{r^n}$, we ſhall have $\frac{A}{r-1} = \frac{B}{r^{\frac{1}{2}}-1}$, from whence will be deduce

duced $B = \frac{r^{\frac{1}{2}}-1}{r-1} \times A$; and in the ſame manner, if the Payments were to be made quarterly, then B would be $= \frac{r^{\frac{1}{4}}-1}{r-1} \times A$, and ſo on.

BUT if we ſuppoſe that a Rent ſhall be paid half-yearly, and that it ſhall alſo be one half of what would be given for an annual Rent, and that the two Rents ſhall be of the ſame Duration; then the preſent Values of the Yearly and half-yearly Rents will be different, for let M and P be the preſent Values of the Yearly and half-yearly Rents, then $M = \frac{A - \frac{A}{r^n}}{r-1}$, and $P = \frac{\frac{1}{2}A - \frac{\frac{1}{2}A}{r^n}}{r^{\frac{1}{2}}-1}$, and dividing both Values by $A - \frac{A}{r^n}$, we ſhall have $M, P :: \frac{1}{r-1}, \frac{\frac{1}{2}}{r^{\frac{1}{2}}-1}$, and conſequently $P = \frac{\frac{1}{2} \times \overline{r-1}}{r^{\frac{1}{2}}-1}$.

THE

THE two laſt Problems bring to my Mind an Aſſertion which was maintained about ſix Years ago in a Pamphlet then publiſhed, which was, that it would be of great Advantage to a Perſon who pays an Annuity, to diſcharge it by half-yearly Payments, each of one half the Annuity in Queſtion; the Reaſon of which was, that then the time of paying off the Principal would be conſiderably ſhortened: I had not the curioſity to read the Author's Calculation, becauſe I thought it too long; ſince which Time I thought fit to examine the thing, and found that indeed the Time would be ſhortened, but not ſo conſiderably as the Author imagined; which to prove, I ſuppoſed a Principal of 2000 *l.* an Annuity of 100 *l.* and the Rate of Intereſt 1.04: in conſequence of which, I found that the Principal would be diſcharged in 41 Years; this being founded on the general Theorem $\frac{A-\frac{A}{r^n}}{r-1}=P$, in which A repreſents

the

the Annuity, P the Principal, r the Rate of Intereſt, and n the Number of Years; now to apply this to the Caſe of half-yearly Payments, let us ſuppoſe that p denotes the Number of Years in which the Principal will be diſcharged, therefore $2p$ will be the Number of Payments; $\frac{1}{2}A$ the Annuity, and $r^{\frac{1}{2}}$ the Rate of Intereſt: which being reſpectively ſubſtituted in the Room of n, A, r, we ſhall have now $\frac{\frac{1}{2}A - \frac{\frac{1}{2}A}{r^n}}{r-1} P =$ but $r^{\frac{1}{2}} - 1 =$ 0.019804, which being ſuppoſed $= m$, we ſhall have $\frac{1}{2}A - \frac{\frac{1}{2}A}{r^p} = mP$, and $\frac{\frac{1}{2}A}{r^p} = \frac{1}{2}A - mP$, or $\frac{50}{r^p} = 10.392$; therefore $\frac{r^p}{50} = \frac{1}{10.392}$, or $r^p = \frac{50}{10.392}$, and $p \log. r = \log. 50 - \log. 10.392 =$ 0.6822709, therefore $p = \frac{0.6822709}{\log. r}$; again, $\log. r =$ 0.0170333, therefore $p = \frac{0.06822709}{0.0170333} =$ 40.05; and therefore the

the Advantage of paying half-yearly would amount to no more than gaining one Year in 41.

QUARTERLY Payments, or half-quarterly, nay even Payments made at every Inſtant of Time, would not much accelerate the Diſcharge of the Principal. Which to prove, let us reſume once more our general Theorem $\frac{A-\frac{A}{r^n}}{r-1}=P$; let us now imagine that the Number of Inſtants in the Year is $=t$, let us further ſuppoſe that s is the Number of Years in which the Principal will be diſcharged, then in the room of A, writing $\frac{1}{t}A$; in the room of r, writing $r^{\frac{1}{t}}$; and in the room of n, writing st, we ſhall have $\frac{\frac{1}{t}A-\frac{\frac{1}{t}A}{r^s}}{r^{\frac{1}{t}}-1}=P$. But it is known, that if t repreſents an infinite Number, ſuch as is the Number of Inſtants in one Year, then

then $r^{\frac{1}{t}}-1=\frac{1}{t}\,log.\ r$, we have therefore $\frac{\frac{1}{t}A-\frac{\frac{1}{t}A}{r^s}}{\frac{1}{t}\,log.\ r}=P$, or $\frac{A-\frac{A}{r^s}}{log.\ r}=P$; let the Logarithm of r be ſuppoſed $=a$, therefore $A-\frac{A}{r^s}=aP$, and $\frac{A}{r^s}=A-aP$, and $r^s=\frac{A}{A-aP}$, which, ſuppoſe, $=\mathfrak{Q}$, then $s=\frac{log.\ \mathfrak{Q}}{log.\ r}$: But it is to be noted, that a repreſents the hyperbolic Logarithm of r, which is, as we have ſeen before, 0,0392207 when r ſtands for 1,04; this being ſuppoſed, the Logarithm of $\mathfrak{Q}$ will be found to be 0,6663794, which being divided by the Logarithm of r, *viz.* 0,0170333, the Quotient will be 39,1 Years; but in this laſt Oepration the Logarithms of $\mathfrak{Q}$ and r, may be taken out of a common Table.

CHAPTER VIII.

Containing the Demonſtration of what has been ſaid concerning the Probabilities of Survivorſhip.

WHAT I call Complement of Life having been defined before *pag.* 2. I ſhall proceed to make uſe of that Word as often as occaſion ſhall require.

HYPOTHESIS.

Let it be ſuppoſed that the Complement of Life AS being divided into an infinite Number of equal Parts repreſenting Moments, the Probabilities of living from A to B, from A to C, from A to D, &c. are reſpectively proportional to the ſeveral Complements SB, SC, SD, in ſo much that theſe Probabilities may reſpectively be repreſented by the Fractions $\frac{SB}{SA}$, $\frac{SC}{SA}$, $\frac{SD}{SA}$ &c. This Hypotheſis being admitted, the

the following Corollaries may be deduced from it.

COROLLARY I.

THE Probability of Life's failing in any Interval of Time AF is measured by the Fraction $\frac{FA}{SA}$.

COROLLARY II.

WHEN the Interval AF is once past, the Probability of Life's continuing from F to G is $\frac{SG}{SF}$, for at F, the Complement of Life is SF, and the Probability of its failing is $\frac{FG}{SF}$.

COROLLARY III.

THE Probability of Life's continuing from A to F, and then failing from F to G is $\frac{SF}{SA} \times \frac{FG}{SF} = \frac{FG}{SA}$.

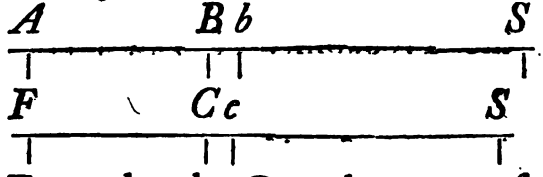
COROLLARY IV.

THE Probability of Life's failing in any two or more equal Intervals of

Time aſſigned between A and S are exactly the ſame, the Eſtimation being made at A conſidered as the preſent Time.

THESE things premiſed, it will not be difficult to ſolve the following Problem.

Two Lives being given, to find the Probability of one of them fixed upon, ſurviving the other.



FOR, let the Complements of the two Lives be reſpectively $AS=n$ and $FS=p$, upon which take the two Intervals AB, $FC=z$, as alſo the two Moments $B\,b$, $C\,c=\dot{z}$.

THE Probability of the firſt Life's continuing from A to B, or beyond it, is $\frac{n-z}{n}$; the Probability of the ſecond's continuing from F to C, and then failing in the Interval $C\,c$, is by the third *Corollary* $\frac{\dot{z}}{p}$, therefore the Probability

bability of the firſt Life's continuing during the time AB or beyond it, and of the ſecond's failing juſt at the end of that Time is meaſured by $\frac{n-x}{n} \times \frac{\dot{x}}{p} = \frac{n\dot{x} - \frac{1}{2}x\dot{x}}{np}$, whoſe Fluent $nx - \frac{1}{2}xx$ will expreſs the Probability of the firſt Life's continuing during any Interval of Time or beyond it, and of the ſecond's failing any time before or preciſely at the end of that Interval.

LET now p be written inſtead of x, and then the Probability of the firſt Life's ſurviving the ſecond, will be $\frac{np - \frac{1}{2}pp}{np} = 1 - \frac{\frac{1}{2}p}{n}$.

FROM the foregoing Concluſion we may immediately infer that the Probability of the ſecond Life's ſurviving the firſt is $\frac{\frac{1}{2}p}{n}$.

BY the ſame method of arguing, we may proceed to the finding the Probability of any one of any Number of given Lives ſurviving all the reſt, and thereby verifying what we have ſaid in *Pag.* 43 and 44.

FINIS.

A VALUATION OF ANNUITIES AND LEASES certain, FOR A SINGLE LIFE. WITH TABLES,

SHOWING

At one View the Value of them, in Years, and the eighth Part of a Year, for every Year of Life, from the Age of 1 to 70, and from thence to the End of Life, for every Period of five Years.

At the ſeveral Rates of 3, 4, 5, and 6 *per Cent.* Intereſt.

By *WEYMAN LEE*, Eſq;
Of the *Inner Temple.*

LONDON:
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Price 1*s.* 6*d.*

To the HONORABLE

Alexander Hume Campbell,

TREASURER,

And the reſt of the

MASTERS of the BENCH,

OF THE

ANCIENT SOCIETY

OF THE

INNER TEMPLE,

THIS

TREATISE,

With all due Deference,

IS

INSCRIBED,

By their moſt obedient

Humble Servant

And Brother,

Weyman Lee.

A VALUATION OF ANNUITIES FOR LIVES, &c.

SOME Time ago, I publiſh'd an Eſſay on the Method of aſcertaining the Value of Annuities, and of Leaſes reduced to Annuities certain, for one or more Lives; and therein I laid down this Poſition as the Groundwork, *viz.* that the even Chance of the Duration of a Life, computed once only for the whole Life, was the ſole true Meaſure of the Value of an Annuity

depending on that Life. But ſince ſome Perſons have made Obſervations on this my Method, and others have formd Valuations of theſe Eſtates, and have defended and ſtill adhere to the Method preſcribed by Dr. *Halley*; I think it will not be amiſs to make ſome Remarks upon them, partly to anſwer and remove the Objections which have been offerd, but chiefly, becauſe it gives me an Opportunity to maintain and further confirm the fundamental Poſition.

Amongſt my Adverſaries Mr. *H. B.* appears to be moſt potent, and indeed the only one who deſerves any ſerious Conſideration; and the Queſtion between him and me is, Whether Dr. *Halley*'s Rule or mine be the righter, for forming an Eſtimate of the Value of Annuities on a ſingle given Life. The Dr's Rule is to find the Value of the Annuity for each Year of the given Life to an 100; which is done, and can be no otherwiſe done, but by computing the Chances of Mortality ariſing in each Year, and deducting the Value thereof out of the Value of an Annuity abſolute for that Year, and then to put together all the Values ſo collected, and the Sum total is the Value of the Annuity for the given Life. Mine is, to find to what Term of Years any Life is equal, or an even Chance may probably continue; and the Value of an Annuity for the given Life will be the ſame as the Value of an Annuity for that Term of Years.

Now if my Rule be a right one, and from the ſlender Objections made to it, and the weak Defence

Defence of the other by this Advocate, and, if this other could be defended, by this Hand it would have been defended, I am fully ſatisfied that it is a right one; this my Rule, and the Tables formd upon it, will have a manifeſt Advantage over any other Rule, and any Tables that have been or can be planned from thence. By the other Rule, if the Value of an Annuity for a given Life, is to be computed, the Computer, on every Variation of the Rate of Intereſt, whether real or on Suppoſition only, muſt, of Neceſſity, calculate the Value of the Annuity for every individual Year, to which the Life of the Perſon nominated may poſſibly continue, for each Year ſeparately and diſtinctly, tho' it ſhould happen to be 80, 90, or an 100 Years; and believe me, thoſe who have tried it, have found this not only a tedious, but very intricate Affair: Nay, this elaborate and perplex Calculation does not attend only on every Change of the Rate of Intereſt, but occurs again in ſome Meaſure on every Age of Life, tho' it differs one Year only, and even tho' the Rate of Intereſt is, or is ſuppoſed to be, the ſame. On the other Side, and upon my Rule, let the Rate of Intereſt vary as often or as much as you will, either in Practice or Imagination, yet the Chance of the Duration of a Life of one and the ſame Age, in one and the ſame Country, will not vary, but perpetually be the ſame, and equal to the ſame Term of Years, and ſuch Term of Years appears in my Tables on the firſt View.

Theſe Tables go farther, and ſhow what is the Value of ſuch Term, that is, what is the Value

of the Annuity for the given Life at the Rates of 3, 4, 5, and 6 *per Cent.* Intereſt. And if we could ſuppoſe that the Intereſt of Money ſhould fall from 3 to $2\frac{1}{2}$, or to 2, which I do not ſuſpect will ever be the Caſe, at leaſt not ſo ſoon as ſome People vulgarly imagin; yet the Term, to which the given Life is equal, will be perpetually the ſame, and will appear in theſe Tables; and, ſuch Term being once known, the Value of ſuch Term will be eaſily found out by the common Tables, which ſhall then be in Uſe, for the Value of Terms for Years, at the Rates of 2 or $2\frac{1}{2}$; for I make no Doubt, but that, if and when thoſe ſhall become the common Rates of Intereſt, ſuch Tables will be conſtructed, for they are very eaſily conſtructed, and will be in every body's Hand.

Mr. *H. B.* begins his Attack on the Eſſay, with ſome Obſervations on the Table for Annuities on a ſingle Life; but ſince I have thought fit, for the Reaſons which I ſhall hereafter mention to alter it, there is no Need to take any Notice of them. From my Table he proceeds to conſider the Rule which I juſt now mentiond, and ſays, he will give a few Reaſons why he thinks the very Foundation I go upon is wrong, and the Rule I firſt lay down not a right one. I am very much obliged to the Gentleman, that he will give but a few Reaſons againſt the Rule; and more ſo, that he has reduced thoſe few to one; but he is Maſter of his own Reaſon. The Rule I have already ſet forth; and the Inſtance I gave of it, which 'tis neceſſary to repeat here, becauſe of frequent References

ferences to it hereafter, was this: The Number of Perſons of the Age of 10, of which we will ſuppoſe *A* to be, by a Calculation made on the *Breſlaw* Table, is 661; the Half of this is 330 ½; and this Number we find between 51 and 52, ſo that *A* has an even Chance to live 41 Years and ſomething over. A Term for 41 Years Intereſt computed at *6l. per Cent.* is in Value 15—12 Centeſimals; therefore an Annuity for the Life of *A*, who has an even Chance to live 41 Years and beyond, muſt be worth 15—12 Centeſimals and above. This Author admits the Premiſſes to be true; *viz.* that *A* has an even Chance to live 41 Years; that a Term for 41 Years is in Value 15—12; but denies the Concluſion, *viz.* that an Annuity for his Life is worth 15—12. His Reaſon for denying this Concluſion runs thus: *That, tho' the Chances of* A's *dying before* 51 *or after* 51 *are equal in Number, yet they are not equal in Value.* He then eſtabliſhes the Truth of this Poſition by a Demonſtration, in two ſeveral Inſtances, and from thence, without further Proof or Allegation, concludes: *Conſequently, an Annuity for* A's *Life, who has an even Chance to live* 41 *Years and beyond, is not worth as much as an Annuity for* 41 *Years abſolute, that is, is not worth* 15—12, *Q. E. D.*

When he ſays, that theſe Chances are of unequal Value, I ſuppoſe he means, that the Annuities, attendant on thoſe Chances for each Year, are of unequal Value; and he cannot well mean any Thing elſe, becauſe in any other Senſe the Poſition is not true; he might have ſaved himſelf the Trouble

Trouble of a Demonſtration, that theſe Chances are of unequal Value, ſince the Annuities attending them are Part in Poſſeſſion, and Part in Reverſion; and, to be ſure, there could not be Occaſion for a Demonſtration, at leaſt, not in two Inſtances, to have ſhown that the Annuities for a Year of a Life muſt be of more or leſs Value, as they are more or leſs removed from the firſt Year's Annuity; for Inſtance, that the preſent Value of an Annuity for the ſeventh Year of a Man's Life is greater than the Value of the like Annuity for his twentieth Year. Admitting then, that theſe Chances, or the Annuities attending them, are of unequal Value, to which, after ſuch an irrefragable Proof as two Demonſtrations, I ſhall certainly ſubmit; Does it follow from thence, that the Rule and the Values produced by it are wrong? This is the Concluſion which he draws; but, as I apprehend, long before he is come at it, for theſe Premiſſes and theſe Demonſtrations do no more prove it, than they do, that the three Angles of an equilateral Triangle are equal to two right ones. He admits that the Rule does not direct to put in more Chances in Number, than the Annuitant has a Right to; and he does not ſo much as aſſert, that it takes in any Chances of greater Value than he has a Right to; and this ought not only to have been aſſerted, but proved too, otherwiſe the Conſequence he draws is a ſtrange one; for the Argument, as it now ſtands, is no better than this: An Annuitant, by this Rule, has no more Chances in Number than he has a Right to; but,

but, in Fact, there are ſome Chances of greater Value than others, without proving or even aſſerting, that any ſuch Chances of greater Value are inſerted in the Computation. Now ſince the Matter in Diſpute is not, whether ſuch Annuitant, by Virtue of the Rule, has more Chances in Number, but whether he has ſome better in Value, than he has a Right to: This is evidently to beg the Queſtion, and not to prove it, notwithſtanding the terrible Sting at the Tail of it; for the Demonſtrations plainly go to one Propoſition, *viz.* to prove this ths or that is the Value of ſuch Chances; and the Argument concludes to another, *viz.* to ſuch and ſuch Chances the Annuitant has a Right, and no other.

However, to help out a lame Argument, for ſuch I muſt call it, as here formd, I will admit, that the Rule does direct to put in, and that the Value produced thereby does include, all thoſe Chances which ariſe on the Life of *A*, before he arrives at the Age of 41; and that the Chances, ſo inſerted, are of greater Value, than thoſe which ariſe after he is paſt 41 Years. But not one Word of Proof is here ſo much as offerd, that theſe greater Values are not the real Values, which an Annuitant on the Life of *A* has the Chance to receive; and in the Eſſay it is aſſerted, and ſo ſtrongly and fully proved, that theſe greater Values, and no other, are the very Values which ſuch an Annuitant has a Chance to receive, that I cannot well add any thing to confirm it.

But

But this Author, inſtead of attempting to invalidate theſe Arguments, or to prove that the Annuitant on the Life of *A*, who has an even Chance to live 41 Years, has not an even Chance to enjoy his Annuity for as many Years as he has an even Chance to live, that is, 41 Years, or to enjoy thoſe Chances which ariſe in thoſe firſt 41 Years of his Life; which he muſt do, before he deſtroys the Foundation of my Rule, and yet I may be confident he never will be able to do; inſtead of this, he very valiantly demonſtrates ſomething, which, to be ſure no Man did ever make any Doubt of, that the Chances, or the Values attending them, where ſome are in Poſſeſſion, and ſome are in Reverſion, are not all of equal Value.

Since this Author is ſo full of, and ſo learned in, the Doctrine of Chances, as he calls it, I will take the Liberty of offering to his Conſideration an Argument or two, in Behalf of my Rule, according to his Way of Thinking. The Value of the Chances in this Caſe, depends on, or is one and the ſame Thing, as, the Value of the Annuities dependant on thoſe Chances. All the poſſible Chances, which an Annuitant has on the Life of *A*, are agreed to be 661, and of thoſe he has an even Chance to enjoy a Moiety, that is, 330 ½. The Annuities which attend thoſe Chances for the whole Life, are Annuities for a Term of 90 Years, it being ſuppoſed poſſible that he may live for ſo many Years; the total Value of them for 90 Years, as I compute, is

16—58

16-58 Centeſimals, and the Value of thoſe in Poſſeſſion for 41 Years is 15-12, and of thoſe in Reverſion 1-46. Since then the Annuitant on the Life of *A*, has an even Chance to 330 ½ Chances on his Life, and the Annuities will attend thoſe Chances; and ſince theſe 330 ½ Chances will, and from the Nature of the Life of Man of Neceſſity muſt, ariſe in the firſt Part of thoſe 90 Years, if they ariſe at all, and 'tis ſuppoſed in the Caſe to be an even Chance that ſo many Chances will ariſe in his Life: He muſt then of Neceſſity have the Annuities attending thoſe very Chances, that is, he muſt have the Annuities for the firſt 41 Years of Life. This Author, or whoever will undertake to anſwer this Argument, muſt ſhow that 'tis poſſible for an Annuitant on the Life of *A* to have the Benefit of all theſe 330 ½ Chances, to which Number of Chances he has without Diſpute a Right or an even Chance to enjoy, unleſs he does enjoy thoſe which ariſe in the firſt Part of Life, and in immediate Poſſeſſion, that is, thoſe of the greateſt Value; and this I am very ſure he cannot ſhow.

Again, Theſe Poſitions are not conteſted: That an Annuity for a Term of 41 Years is in Value 15-12; and that an Annuitant for the Life of *A* has an even Chance that *A* lives for 41 Years. 'Tis a Maxim made uſe of by this Author, and is certainly a true one, that in eſtimating the Value of Annuities for a Life all the poſſible Chances of Life muſt be computed. From hence I argue thus: To eſtimate the Value

of an Annuity for the Life of *A*, we muſt compute all the poſſible Chances on the Life of *A*; the Chance that *A* lives for 41 Years is an even Chance, and conſequently is one of the poſſible Chances on the Life of *A*; therefore to eſtimate the Value of an Annuity for his Life we muſt compute this even Chance. The even Chance on this Life is to a Term for 41 Years. A Term for 41 is in Value 15-12. Therefore the Value of an Annuity for this Life is 15-12.

This Author, having by this and by no other Argument demoliſhd my Rule, even to the very Foundations, as he expreſſes himſelf, proceeds to conſider Dr. *Halley*'s Rule; and, having given it us, pronounces that 'tis moſt certainly right, my Objections notwithſtanding; and, to be even with him, I ſay, notwithſtanding his Defence, that 'tis certainly wrong. One of my Exceptions to his Rule, and which this Writer's Obſervations on it make neceſſary to repeat, is this: That the Value produced by his Rule is not the Value of an Annuity for the Life of *A*, but will be the Value of 90 ſeveral Annuities or Grants made to 90 ſeveral Perſons, ſuppoſe to N. 1, to N. 2, and ſo on to N. 90, formd in this Manner, *viz.* to N. 1 for a Year, if he ſhall live from 10 to 11 Years of Age; to N. 2 for a Year, if he ſhall live from 11 to 12 Years Age; and ſo on ſucceſſively to N. 90 for a Year, if he ſhall live from 99 to an 100 Years Age. I then affirm, that the Value of an Annuity for the Life of *A* abſolutely, or for 90 Years, if *A* ſo long lives,

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is of a different and much greater Value than the Intereſt of N. 1, N. 2. and ſo on to 90, all put together. To this he ſays, that it woud take up too much Time to anſwer all the Arguments I have brought to ſupport this Aſſertion. This ſurely is a very ſmall Reaſon for not attempting to anſwer any of them, or the moſt material of them; and ſom People will be apt to ſurmiſe that he wanted ſomething elſe rather than Time to anſwer them. However, to be ſure it will be neceſſary for me to ſpend my Time in repeating and defending them; ſo that, ſince he has left them quiet and unanſwerd, I ſhall e'en leave them, in *ſtatu quo*, undefended.

He does indeed afterwards make an Attempt to prove, that an abſolute Annuity for this Life, which I contend to be in Value 15-12, is of no greater Value than theſe ſeveral Annuities or Grants, which I ſuppoſe to be made to 90 Perſons for 90 Years, when put together, that is, not more than 13-44. He ſays then, that 90 ſeveral Annuities granted to *A*, in this manner; the firſt Annuity for one Year, if he ſhall live from 10 to 11 Years Age; the ſecond for one Year, if he ſhall live from 11 to 12 Years Age, and ſo on ſucceſſively to a 90th Annuity for one Year, if the Nominee ſhall live to be an 100 Years old; will be of equal Value with one Annuity granted to him for his Life abſolutely: For the Chances of receiving the Annuity will be the ſame in both Caſes, becauſe the Chances of receiving it depend on the Chances of his Vitality. Here he alters

the State of the Caſe intirely from what I gave; for whereas I ſuppoſe theſe ſeveral Annuities or Grants for 90 Years, determinable at the End of each Year, to be made to 90 ſeveral Perſons on their ſeveral Lives, he ſuppoſes theſe ſeveral determinable Annuities to be granted to one and the ſame Perſon, and on one and the ſame Life; which Alteration manifeſtly varies the State of the Chances. The Difference between the two Caſes is evidently this; that in the Manner I ſtated it there are as many Chances of Mortality as there are Lives, *viz.* 90; but, in the Caſe as he ſtates it, there is but one Chance of Mortality, for there is but one Life in the whole, *viz.* the Life of *A.*

He aſſerts farther, that theſe ſeveral Annuities, as he calls them, *will be of equal Value, whether they are granted by one Perſon to one Perſon, or by* 90 *ſeveral Perſons to one Perſon; and all equal to an abſolute Annuity for a Life.* That they are ſo in the firſt Caſe I admit, becauſe the Grants amount to an abſolute Annuity for a Life; but that they are ſo in the latter Caſe, as well on the Part of the ſeveral Grantors as of the Annuitant, is a Point at leaſt doubtful; and I ſhall go farther and affirm, that they are not in all Inſtances equal, and that the Reaſon he gives to prove their Equality, *viz. That the Chances for paying and receiving it muſt in both Caſes be the ſame*, is a Fact not in all Caſes true, and therefore does not make good his Aſſertion. In the firſt Place, if ſeveral Grants were made to one and the ſame Perſon, *viz. A.*

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by 90 diſtinct Perſons in the manner here ſuppoſed, it might be a Queſtion, whether ſuch Grants were properly an Annuity, or a Grant only of ſo many ſeveral Sums in Groſs made by ſo many diſtinct Perſons. With Regard to *A*, the ſeveral Sums ſo granted, when put together, woud amount to ſomething which woud be equivalent to an Annuity for his Life; but *quoad* each ſeparate Grant, they ſeem to be no other than a Grant of ſo many ſeveral Sums in Groſs. If we conſtrue them as an Annuity for the Life of *A*, a Diſpute woud then ariſe between *A* and the ſeveral Grantors, whether the greater Sum, *viz.* 15-12, or the leſs Sum, *viz.* 13-44, was to be paid to the Grantors in proportionable Parts. *A* on his Side may allege, that the Grantors, on their Ingagements for the reſpective Sums to be paid *in futuro*, are to receive no more than 13-44 in the whole, and all put together, becauſe *quoad* them the preſent Value of the Chances to pay them, ſeverally and diſtinctly conſiderd, do amount to no more. The ſeveral Grantors on the other Side may allege, that *A* on the Purchaſe of ſuch Grants is to pay for them in the whole, and all put together, 15-12, becauſe *quoad* him the preſent Value of all the Sums put together which he has a Chance to receive, by the Combination of their ſeveral Grants, amounts to that greater Sum: And, believe me, ſuch their Allegation is founded on very juſt and good Reaſons. If theſe Grants, ſuppoſed to be made by 90 ſeveral Perſons, are only ſo many Sums in Groſs,

Groſs, they are not pertinent to the preſent Purpoſe; and, if we conſtrue them to amount to an Annuity for Life, 'tis diſputable at leaſt which of the two Sums is to be paid, the greater or the leſs; and it may be Time enough to diſcuſs that Controverſy when any ſuch Grants ſhall be made, which is not very likely ever to come to paſs; this imaginary Caſe therefore will avail nothing to determin the Point in Diſpute, but it muſt be left to be decided by other Arguments and upon other Topics.

I affirm then, in the ſecond Place, that this Author's Reaſon, *viz.* that the Chances of paying or receiving in both theſe Caſes are the ſame, do not prove that the Annuities in each are of equal Value. In this Matter, the Chances of not paying, or not receiving, are to be taken into the Conſideration as well as thoſe of paying and receiving. This being premiſed, I ſay, that in the Caſe of a ſingle Grantor of an Annuity for the Life of *A*, he has only one Chance during the whole 90 Years that he ſhall be exempted from paying, but the ſeveral Grantors have, each in their reſpective Years, a Chance for ſuch Exemption; conſequently the Value of the ſingle Chance of not paying in the firſt Caſe is not the ſame as the Value of the accumulated Chances in the latter; therefore the Value of the Annuities attending is not equal. Each of theſe Grantors of the Sums in Groſs or annual Payments, on their Grants, muſt be allowd and have a Deduction for ſo many Chances as each of them has of not

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paying the Sum at the End of each Year, that is, for ſo many Chances of Mortality as ariſe on the Life of the Annuitant in each Year reſpectively. And why is this done? Becauſe the Chance of Mortality *quoad* each of them ariſes in each Year, and is confined to each Year reſpectively. But in the Caſe of a ſingle Grantor, a Grantor of an Annuity for a Life abſolutely, or a Combination of Grantors who ſhall make ſo many Grants as to be equal to an Annuity for a Life, this Chance of Mortality, and conſequently of not paying, does not ariſe in that Manner, but does, and from the Nature of the Life of Man of Neceſſity muſt ariſe at once, and once only; and conſequently on the Purchaſe an Allowance and Deduction can be made for one Chance only, ſince it can ariſe but once. Now that theſe Chances of Mortality, for which an Allowance and Deduction is to be made, in the Caſe of the ſeveral Grantors, when all put together, are of greater Value than the ſingle Chance of Mortality in the Caſe of the ſingle Grantor is evident from hence; becauſe the Chances in the firſt Caſe ariſe, ſom in the firſt Year, ſom in the ſecond, and ſo on ſucceſſively for 90 Years; whereas the one Chance of Mortality in the latter Caſe does not ariſe till the even Chance of the Mortality of *A*, that is, not till the End of 41 Years; and Chances in immediate Poſſeſſion are of much greater Value than one or the like Number of Chances in Reverſion after 41 Years, as this Author has very learnedly demonſtrated.

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I have ſuppoſed here, that a ſingle Grantor of an Annuity for a Life has but one Chance of the Mortality of his Annuitant throughout the whole 90 Years, and moſt certainly he has not. That every Man ſhall die once, and no Man more than once, muſt be admitted; but 'tis abſurd to ſay that a Perſon has more than one Chance to ſuffer that which 'tis impoſſible he ſhoud ſuffer more than once. However, if it be inſiſted on, in the Caſe put of 90 ſeveral Grants to *A*, that the Chance of receiving the Annuity is the ſame as in the Caſe of an Annuity abſolutely for his Life, and it ſeems to be the better Opinion, then *A* has in both Caſes the ſame Chance of Mortality againſt him, and that is one only: Then it will be inſiſted on by the other Side, that *A* having by ſuch a Combination of Grants put himſelf into ſuch a Condition, that he has only a ſingle Chance of Mortality againſt him, and of not receiving his Annuity, that then for theſe accumulated Grants he muſt pay the ſame Sum as he ſhoud do for an abſolute Annuity for his Life, that is, the greater Sum.

Hitherto we have been ſpeaking of and conſidering what woud be the Caſe of *A*, whom we have ſuppoſed to be 10 Years old only, on a ſingle Grant of an Annuity for his Life abſolutely, and on 90 ſeveral Grants to him by 90 Perſons, determinable by his Death within any one Year of 90 Years: And, if the Point in Diſpute remains there doubtful, the Inſtance which follows will put the Matter beyond all Doubt. We will

ſuppoſe

ſuppoſe then that *B* is 80 Years old, that there are 20 Years of poſſible Life remaining to him, that he has from one Perſon an abſolute Annuity for his Life, and has likewiſe 20 ſeveral Grants made to him by 20 ſeveral Perſons determinable in the ſame Manner as in the former Caſe. The even Chance of the Duration of the Life of *B*, Computation being made of ſuch Chance by the *Breſlaw* Table, is 4 Years, or hardly ſo much; and an Annuity for 4 Years, Intereſt ſtated at 6 *l. per Cent.* is 3-46, an Annuity for the Life of *B* abſolutely is therefore in Value 3-46. An Annuity or Grant for the Life of *B*, when made by 20 ſeveral Perſons for 20 Years, determinable in the Manner aforeſaid, is equal to a Term for four Years and an Half, and is in Value 3-84, all which Facts are admitted by this Writer: From whence 'tis evident, as evident as Numbers or Figures can make it, that the Annuities in both Caſes are not equal. 'Tis true likewiſe, and as demonſtrable, in the Caſe of *B*, that the Chances of receiving or paying are not the ſame in both Inſtances, unleſs it be the ſame Thing to have a Chance of receiving 3-46, as 3-84; or *B* have a Chance of receiving the Annuity for his Life, after he is dead and gone.

He adds farther, that the Chances of Vitality cannot be increaſed or decreaſed by the Manner of granting the Annuity: But, I think, he ſhoud rather have ſaid, that they ought not, or ſhoud not be varied by the Manner of granting, for

'tis viſible, that in Fact they are varied in the Caſe of *B*; for the Chance of Vitality on the ſingle Annuity for his Life is only four Years, and very barely that; but on the Annuity by the accumulated Grants the Chance is four Years and an Half; and 'tis a Contradiction to common Senſe to ſay he has any Chances of Vitality left on the accumulated Annuity, when on the ſingle abſolute Annuity they are all determind and gone.

I woud here recommend it to this Writer to reconcile theſe Matters, but that I am afraid he will be ſick of his reconciling Function, when he finds, as he will do in an Inſtance herein after mentioned, how little Succeſs he meets with in ſuch Undertakings. I ſhall chooſe rather to obſerve that this Argument, as weak as it is, is the only one which this Author has offerd to maintain the Juſtice of Dr. *Halley*'s Rule, for making a Valuation of theſe Annuities; and proceed to conſider what may ſeem to be alleged by him, in Anſwer to my main Exception to the Dr's. Rule.

The Exception I mean, and which is the plaineſt and ſtrongeſt againſt the Dr's. Rule, may be reduced to this ſhort Syllogiſm: Any Rule for computing the Value of Annuities for a Life, which produces ſuch a Value as that one and the ſame Life ſhall be equal to a different Term, as the Rate of Intereſt varies, is a falſe Rule: This Rule does neceſſarily produce ſuch a Value: Therefore is a falſe one. The firſt Propoſition is ſo clear and obvious a Truth, that this Author

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does not offer to contradict it, nay, he goes farther, and by his own Poſition juſt mentioned, *viz.* That the Chances of Vitality cannot be increaſed or decreaſed by the Manner of granting an Annuity for a Life, he does effectually confirm it; for if, under the Direction of the Rule, ſuch Sums are neceſſarily produced as the Value of an Annuity on a Life, as that the Term to which ſuch Life is equal, or the Chance of the Duration of ſuch Life, ſhall vary as the Rate of Intereſt varies; moſt certainly this is to make the Manner of granting the Annuities, and the Rate of Intereſt at which they are granted, to govern and to increaſe or decreaſe the Chance of Vitality on that Life. My ſecond Premiſe I demonſtrate by ſhowing that Mr. *Richards*'s Tables, which were conſtructed by that Rule, do in Fact exhibit ſuch a Value of theſe Annuities. This appears by an Inſpection into his Tables, where the Value of an Annuity for a Life of 12 Years old, when Intereſt is computed at 4 *l. per Cent.* is 17-20 Centeſimals, which is equal to a Term of 29-3-00; and when computed at 8 *l. per Cent.* is 10 67 Centeſimals, which is equal only to a Term of 25—. To ſhow that theſe Values do not proceed from any Error or Defect in the Operation made by Mr. *Richards*, but are a neceſſary Reſult from the Rule, and conſequently the Rule a falſe one, I advance theſe two Propoſitions: That if a certain Value and Term correſponding are given, and out of that Value a proportionate Part be deducted; firſt, that the Term correſpond-

ing to the Value remaining will not bear the ſame Proportion to the Value remaining, as the original Term does to the original Value; ſecondly, that the Term correſponding to the Value remaining, after a Deduction at one Rate of Intereſt, will greatly differ from the Term correſponding to the Value remaining, after a Deduction made at another Rate of Intereſt: To which I add, that at 4 *l. per Cent.* the Difference of the Term is the leaſt, and at 8 *l. per Cent.* the greater of the two. To prove theſe Poſitions I have ſubjoind a Calculation which demonſtrates them. Theſe Poſitions and the Calculation this Author has inſerted at length in his Obſervations, *p.* the 26*th*, without taking Notice what introduced them, or how they are applied; however, he admits the Truth of them, and that from thence it neceſſarily follows, if the proportionate Part deducted out of a given Value be an 100th Part only (my Calculation being made on an half Part of the Value) yet that the Term correſponding to the Value remaining, after the Deduction, muſt be different at different Rates of Intereſt. But he cannot agree that this is a Demonſtration that the Rule is wrong. I did not lay down theſe Poſitions immediately and directly to prove that the Rule is wrong, but to prove that the Values given in the Tables were a neceſſary Reſult from the Rule; and they do demonſtratively prove it, and he admits they do: And then indeed, if the Premiſſes are true, and of one of them he takes no Notice or offers to oppoſe, and the

the other he cannot contradict without contradicting himſelf; the Conſequence, that the Rule is a wrong one, will follow very cloſe at the Heels of them, for the Concluſion is a neceſſary and certain one, as neceſſary and certain as any in *Euclid*: For if the Rule does neceſſarily produce Values, which are certainly falſe, the Rule which produces them muſt neceſſarily be falſe likewiſe.

This Author however, notwithſtanding this ſtrong Proof, I may ſay Demonſtration, of the Falſity of the Rule, ſeeming to offer ſomething againſt it, I ſhall conſider it. He ſays then, *that the Rule does not bid you deduct from the Value a Part proportionate to a Part deducted from a Term, but bids you deduct from the Value a Part proportionate to the Number of Chances of Mortality within that Year, which are Things widely different.* 'Tis in no ſort material what are not the Directions of the Rule, nor whether they are Things widely different from the real Directions; however, I will admit this Allegation; but this Writer, I ſuppoſe, becauſe he coud ſay nothing to the real Queſtion, had a Mind to raiſe an Imagination, and palm it upon the Reader, that the Queſtion was, whether this or that was the Direction of the Rule, and how far the one might differ from the other. Be that as it will, in the *Eſſay* 'tis ſaid, and here repeated by this Writer, that one of the Directions of the Rule is this, *viz.* from the Value of an abſolute Annuity for a Year to deduct a Part of ſuch Value proportionate to the Number of Chances of Mortality ariſing in that Year.

Year. Now this is ſo far from a Reaſon for not agreeing with me, that it ſhows, whether he ſees it or not, or will acknowledge it, that he does or muſt agree with me, and proves for me that my Syllogiſtical Reaſoning demonſtrates the Falſity of the Rule. The Matter in Controverſy is, whether this Rule for finding the Value of Annuities on a Life be a good one; and, having denied it, amongſt other Exceptions I alleged this as one, that it directs ſome Deductions to be made in ſuch a Manner as produces ſuch a Value of theſe Annuities as make ſomething to happen on the Life of Man, which is abſolutely inconſiſtent with any Thing that ever can happen in the Life of Man. To this the Anſwer is no more or better than this: That the Rule is a good one, for that it directs theſe Deductions to be made in the Manner I allege, that is, in a Manner which produces thoſe abſurd Values. If the Matter were here freſh, and no Objection had ever been made to the Rule, this would have been a poor Argument to prove its Goodneſs; for 'tis in Effect begging the Queſtion, and ſaying that the Rule and the Directions are ſuch and ſuch, for it does no more than repeat one of the Directions of the Rule, and tell us that it differs widely from another that is no Direction of the Rule, without offering one Word in Juſtification of it. But now, when he is attempting to anſwer a Treatiſe, in which are many Objections to the Rule, and to that very Direction in particular, and in which 'tis alleged, that ſuch and ſuch abſurd Values in certain

certain Tables were produced by following that Direction, and demonſtrated that thoſe Values ſo produced were a neceſſary Reſult from ſo following it: When he tells us that the Rule bids us make the Deductions in the Manner alleged in the Exception; this manifeſtly confirms the Exception, for it admits the Verity and Juſtice of the Allegations; and, if thoſe are true, I'm very confident that the Concluſion is right, that the Rule is not a good one.

Another Objection to the Rule mentiond in the *Eſſay*, and which this Writer's Obſervations upon it make neceſſary here to repeat is this: That according to ſuch Rule the Value of an Annuity for a Life of ten Years Age is 13-44 Centeſimals, which is equal to a Term of 28-1-00; and the Value of an Annuity for a Life of 80 Years Age is 3-84 Centeſimals, which is equal to a Term of 4-2-00; but from the *Breſlaw* Table, from whence this Calculation of theſe Values was made, it appears that the even Chance of the Duration of the Life of 10 Years Age is 41 Years, and the even Chance of the Duration of the Life of 80 Years is not full 4 Years. From thence it was argued: That an Annuity for the Life of a Perſon aged 10 Years ſhoud be in Value 13-44, that is, ſhoud be equal to a Term of 28-1-00 only, when his Life on the even Chance of Duration is equal to 41 Years and above; and at the ſame Time that an Annuity for the Life of a Perſon aged 80 ſhoud be in Value 3-84, which is equal to a Term of

4-2-00,

4-2-00, when his Life on the even Chance of Duration is equal only to a Term of 4 Years: This is there ſaid to be ſo palpable a Contradiction to common Senſe, that nothing can maintain the Rule by which thoſe Values were produced. The Jet of the Argument, lies in this, and on this the Contradiction is grounded, *viz.* That the ſame Rule ſhoud in the Operation have ſuch different Effects on the younger Life and the elder Life as to depreſs the Chance of Vitality of one below the even Chance, and exalt the other above it; which, in Effect and in other Words, is as much as to ſay, that in the one Caſe the Annuity will not have a Chance to continue ſo long as the Life will have a Chance to continue, and in the other Caſe will have a Chance to continue longer than the Life does: And, notwithſtanding the learned Labors of this Author to reconcile this Matter, I think, and I believe moſt others will think, that it remains ſtill in the ſame State of Inconſiſtency.

Let us now examin his Way of Reaſoning here. He admits the Values of the Annuities, and the Chances of the Duration of the Lives, reſpectively, to be ſuch as I have ſtated them; but a Repetition of thoſe Computations, merely and by itſelf, ſure enough does not prove either that the Values produced are the true Values of the Annuities, or that the Rule by which they were computed is a right one. This is neither better nor worſe, than firſt to ſuppoſe and take for granted, that theſe Values thus produced are the true

true ones, and then to infer that the Rule by which they were computed is a right one; or *vice versâ*, to ſuppoſe and take for granted that the Rule by which they were computed is a good one, and from thence to infer that the Values are true ones; but this will not have much Weight with thoſe who deny both that the Rule is a good one, and that theſe Values are the true ones.

However, to juſtify the Value given on the younger Life he urges, *p.* the 33*d*, that, when all the poſſible Chances are conſiderd, they will reduce it to an Equality in Value with a Term of 28-1-00. I obſerve here, that he ſpeaks only of the poſſible Chances of Vitality, and not of the Value of thoſe poſſible Chances; and I ſay, that all the poſſible Chances of Vitality have been conſiderd, and upon a Computation of them, it appears that 'tis an even Chance, that the Life will have Continuance for 41 Years, and this he will and does admit; therefore 'tis incumbent on him to prove that 'tis not an even Chance or Probability that the Annuity, which is to continue for the Life, will have a Duration for the ſame Term, as 'tis an even Chance or Probability that the Life will have a Duration, that is, for 41 Years: And until he does this, which I deſpair of ever ſeeing done, for indeed 'tis impoſſible to be done, the Difficulty will remain.

To juſtify the Value given on the elder Life, which ſeems to be the harder Piece of Work, he gives us, *p.* the 33*d*, a Computation of ſom of the poſſible Chances on ſuch a Life, and refers us to *p.* the 18*th* for a like Computation on ſom of

the like poſſible Chances on the younger Life; and then concludes: *That it will be no Contradiction to common Senſe to ſay, that the Values of theſe two particular Annuities, produced by the Dr's Rule are the true Values of the Annuities on theſe Lives reſpectively: Becauſe in eſtimating the Value of an Annuity on a given Life all the poſſible Chances muſt be computed; and the Chances in the Caſe of the elder Life have a different Operation from what they have in the Caſe of the younger Life.* If he means here no more than that all the Chances of poſſible Life muſt be computed, as his Words import, I ſhall readily agree with him, becauſe we cannot compute the Value of an Annuity for a Life without eſtimating firſt the Chance of the Duration of that Life, and we cannot rightly eſtimate that Chance without computing all the poſſible Chances of that Life. When that is done according to the Rule laid down in the *Eſſay* for eſtimating the Value of Life Annuities, a Moiety of all the poſſible Chances gives us the even Chance of the Duration of the Life, the even Chance of the Duration of the Life gives us the Term to which the Life is equal, and that gives us the Value of the Annuity. But if he means here that the VALUE of all the poſſible Chances on the Life are to be computed, and the poſſible Chances of a Life and the Value of thoſe poſſible Chances are quite different Things, and will produce Annuities of very unequal Value; and this latter he muſt mean, if he intends to ſupport the Dr's Rule, for the Direction of that Rule is to take the VALUE of all the poſſible

Chances

Chances of Life; then he begs the Question, according to Custom, instead of proving it. The Objection to the Rule was this; that it produces such Values of Annuities, as that in the one Case Case they necessarily suppose the Chance of the Duration of the Annuity to fall short of the even Chance of the Duration of the Life, and in the other to exceed it; and these Values of Annuities are in the *Essay* asserted, and most People I believe will agree, to be absurd ones. 'Tis there demonstrated, that these Values of Annuities are produced by the Rule, and particularly by its directing, or because it directs, that the VALUE of all the possible Chances on Life be computed; and he concludes here that the Rule is a good one, without more saying, than that 'tis a good one, because it directs the Computation of the Value of such Chances to be made in a particular Manner, which Manner necessarily produces such a Value of these Annuities as are manifestly absurd ones.

I have said that the possible Chances of a Life and the VALUE of those Chances are different Things, and so most certainly they are; and tho' this Author does not see it, or may not be ready to acknowledge it, that there is in Truth the Matter in Controversy, whether the Computation is to be made by the possible Chances of a Life, or by the Value of those possible Chances. I say 'tis to be made in the first Manner, and that such Computation will give us the Duration or Chance of the Duration of the Life; that the Duration of the Life is the Measure of the Duration of the Annuity depend-

ing on that Life, and the Duration of the Annuity the Meaſure of the Value of it. On the other Side, if a Computation be made of the VALUE of ſuch poſſible Chances, that is done, and can be done no otherwiſe, than by computing the Value of the ſeveral Annuities attending on ſuch poſſible Chances in every Year of Life, and through the whole Life, and then aſſuming that thoſe ſeveral Annuities put together conſtitute the Value of the Annuity for the the Life. Now this does directly and obviouſly make the Chance of the Duration of the Life to depend on the Value of the Annuity; for, in every Computation which has been made of the Value of Annuities on Lives conſtructed in this Manner, we have ſeen that the Fact is ſuch, that in all Inſtances where the Intereſt is varied, and conſequently the Value of the Annuity varied, there the Chance of the Duration of one and the ſame Life is varied likewiſe, and that the one increaſes or decreaſes when and in Proportion as the other does; and yet 'tis moſt undoubtedly true, and this Author aſſerts it, over and over again, that the Value of the Annuity is not to increaſe or decreaſe, or in any Shape to have an Influence on the Chance of the Duration of the Life, and yet in Practice he generally contradicts it.

He adds farther, that the poſſible Chances in the Caſe of the elder Life have a quite different Operation from what they have in the other Caſe of a younger Life, and from thence would infer

that

that the Values of the Annuity on theſe two Lives are or may each be right; but this ſurely is a ſtrange Inference, for he gives that as a Reaſon for the Rectitude of the Rule and for the Truth of the Values, which I have given, and moſt certainly is a good one, for Falſity of both. When I conſiderd the Value of the Annuity on theſe two Lives as produced by one and the ſame common Rule, I ſaid that it coud not be a good one for eſtimating the Values, and gave this Reaſon for it, *viz.* becauſe in the Operation upon it the Effects in the Caſe of the younger Life, and in the elder were ſo different, as to depreſs the Chance of Vitality on one Life below the even Chance, and to exalt the other above it. The Anſwer he gives to this is no more than a bare Repetition of the Grounds of my Objection to the Rule and the Values in theſe Inſtances produced by it, with ſom ſmall Difference in the Expreſſion, *viz.* the Rule is a good one, and the Values of theſe Annuities are right; becauſe the Value of the poſſible Chances, which the Rule directs to be computed, have a different Operation in the younger and in the elder Life. This, I ſay, is and will remain an unanſwerable Objection to the Rule, until it can be ſhown, and I may be confident it can never be ſhown, that the Value of theſe poſſible Chances, or rather the Value of the Annuities attending thoſe Chances, can in ſom Inſtances abridge the Chance of Vitality ſo as to make it leſs, and in other Inſtances inlarge it ſo as to make it greater than the even Chance: Or, as this Author expreſſes

preſſes himſelf, that the Manner of granting theſe Annuities, whether on a younger or on an elder Life, can increaſe or decreaſe the Chance of Vitality. That the Operations on the Value of the poſſible Chances on the younger and the elder Life, in their Way of computing them, have different Produces is a Thing viſible, and is the Matter of the Objection or the Grounds of it; and to anſwer this the Author ſhoud have ſhown ſuch a Difference between the Nature of the Life of the young Perſon, and the old one, as ſhoud reconcile this Difference in the Produces; but that being impoſſible to be done, as moſt certainly it is, for the Chance of Vitality of one to 41 Years is the very ſame Thing, and collected in the ſame Manner, as the Chance of Vitality of the other to 4 Years, he contents himſelf with repeating the Subſtance of the Reaſon and Grounds of the Objection, and there leaves the Matter.

However, to be more particular, he attempts to ſhow that 13-44 may be the Value of an Annuity for a Life of ten Years Age, and 3-84 the Value of one for a Life of 80, tho' neither of them be conſiſtent with the even Chance of Vitality of the reſpective Lives, which he muſt and does admit. For this Purpoſe he makes a Computation of the Value of the Chance which *A*, for Inſtance, ſuppoſed to be 10 Years old, has whether he ſhall die in the firſt Year, or live to be 90 Years old, and the Sum ariſing is 1-59. He makes the like Computation of the Value of the like Chance on the Life of *B*, for Inſtance, ſuppoſed

poſed to be 80 Years old, and the Sum ariſing there is 4-5. If we inquire how the Computations of the Values of theſe particular poſſible Chances on theſe two Lives made by this Author, and his Reaſoning upon them, are to be applied; I can make nothing other or further of them than this.

The Dr's Rule for eſtimating the Value of Annuities a Life may be a right one, altho, in computing the Value of the poſſible Chances of theſe two Lives, the Produces are ſo different as not to coincide with the even Chance of Vitality, which is the Point he undertakes to prove; for here is a Rule, he ſays, of his own, for computing ſom of the particular poſſible Chances on theſe two Lives, and on the Computations made thereby it appears that one is in Value 1-59, and the other 4-5. Very true; but theſe two Values do in Fact coincide with the even Chance of Vitality, as we ſhall ſee preſently, and he himſelf ſays they do, and takes ſom Pains to prove it. The Argument then will ſtand thus: This Obſervator's Rule for computing the partiular Chances on theſe two Lives is a good one, as I do admit it is; becauſe the Values produced by the Rule do coincide with, and are compared by the even Chance of Vitality; therefore Dr. *Halley*'s Rule for computing the Value of the Chances for the whole Life (or of an Annuity for the whole Life) of theſe two Perſons is a good one, which produces ſuch Values as do not in either of them coincide with the even Chance of Vitality. I ſhoud ſpeak too

favourably

of theſe two Computations, and the Application of them to the preſent Purpoſe, if I ſhoud ſay of them only that they prove nothing towards maintaining the Doctor's Rule; for ſom People will be apt to ſurmiſe that they prove rather the contrary, that no Computations on the poſſible Chances of Life, or the Values of them, are right but what do coincide, or at leaſt are conſiſtent, with the even Chance of Vitality of Life.—

To ſupport ſuch Surmiſe of theirs, they make Uſe of the two Computations made by this Writer to ſhow that the Sums 1-59 is the Value of one of the Chances, and 4-5 the Value of the other; but, ſince they are both formed by the ſame Pattern, I will mention one only—He ſays, then; *That the Number of Perſons dying in the firſt Year of the Life of A is* 8, *the Number of Perſons living of the Age of* 90 *is* 8; *conſequently it is an even Chance whether A is one of the firſt* 8, *or one of the laſt* 8; *that is, the Chances of A's dying the firſt Year, or living to* 90, *are equal. If he dies the firſt Year, he loſes the Value of an Annuity for* 41 *Years, which is worth* 15-12 *Centeſimals; whereas, if he lives to* 90, *he wins the Value of an Annuity for* 39 *Years, in Reverſion after a Term for* 41 *Years, which is worth no more than* 1-59 *Centeſimals.* I obſerve here that this Author, as great a Maſter as he is of Numbers and Figures, is guilty of a Miſtake in computing the Value of the reverſionary Intereſt of the 39 Years, and ſtating it at 1-59, when 'tis only 1-38; for the whole

whole Term of 80 Years is, in the common Tables, ſet at 16-50 Value; and, ſince the Intereſt of 41 Years in Poſſeſſion is 15-12, the Intereſt of the 39 Years in Reverſion can be no more than 1-38. But to let that paſs, for in the preſent Argument 'tis not at all material which is the truer Value; and we will ſuppoſe 1-59 to be the right one; I admit then that the Number of Chances are equal; that the Chance here is an even one; and that 1-59, the Sum produced, is the right Value: But it happens very unluckily that this Argumentation, when applied to the preſent Purpoſe, does directly and evidently prove my Poſition, and deſtroy the other. Mine is, that the even Chance of the Duration of the Life, computing one Chance only in the whole Life, is the Meaſure of the Value of the Annuity for that Life; and Dr. *Halley*'s and this Author's is, that the Value of the Chance for each Year of the poſſible Life, computing as many Chances as there are Years in the poſſible Life, is ſuch Meaſure. Now in the Caſe here ſtated 'tis ſuppoſed, as the Foundation of the Truth of the Value produced, that the Chance is an even one; and in the Computation itſelf 'tis taken for granted, that the Chance of the Duration of the Life is the real Meaſure of the Value of the Annuity for the Life. 'Tis here laid down that *A. by his Death in the firſt Year will loſe an Annuity for* 41 *Years which is in Value* 15-12; but by his Death at *That* Time he can loſe nothing but an Annuity for his Life, for he can

loſe nothing but what he has a Right or a Chance to have, therefore ſuch Annuity muſt be equal to a Term for 41 Years, and be in Value 15-12. And ſince the even Chance of the Duration of the Life is a for a Term of 41 Years, and a Term for 41 Years is in Value 15-12, and the Loſs by his Death in the firſt Year is ſuch a Term and ſuch a Value: It follows from thence, that the Chance of the Duration of the Life is here made the Meaſure of the Value of the Annuity for the Life, according to my Poſition.

I cannot forbear obſerving here, how naturally and eaſily this Author falls into the Notion, that the Chance of the Duration of a Life is the Meaſure of the Value of an Annuity on ſuch Life; for his Reaſoning is formed on Purpoſe to prove that ſuch Chance is not the Meaſure, and that 15-12, the Sum produced by ſuch Rule, is not the Value; yet even here he ſuppoſes that Sum to be the true Value of the Annuity, tho' at the ſame Time he makes Uſe of it as the Medium to prove that 'tis not the true Value. If then 1-59 be the Value of this particular Chance on ſuch Life, and for that Reaſon, *viz.* becauſe 'tis an even Chance; I don't ſee why I may not conclude that 15-12 is the Value of an Annuity for the younger Life, and 3-46 of an Annuity for the elder Life, beforementioned: For the even Chance is that the younger Perſon does not die before the End of 41 Years, and a Term for 41 Years is in Value 15-12; and the even Chance is that the elder Perſon dies in 4 Years, and a

Term

Term for 4 Years is in Value 3·46. On the other Side, and from hence, I think I may infer that the Values of the Annuities on theſe Lives produced by Dr. *Halley*'s Rule muſt be falſe, ſince one falls greatly ſhort of the Value of the even Chance of the Duration of the Life, and the other exceeds it. Therefore, upon the Whole, I recommend it to the Author of theſe Obſervations, and leave it with him, to anſwer his own Arguments, as well as mine, for my fundamental Poſition; that the ſingle Chance of the Duration of a Life, computed once for all his Life, is a good Rule, and the only Rule, to compute the Value of an Annuity depending on that Life.

I muſt, in the next Place, take Notice of the Remarks made on the Eſſay made by Mr. *Ritbards*, often therein mentioned, were it only to let him know that I have read them; but that I deſpiſe the foul-mouthed and ſcurrilous Language which he has ſo plentifully beſtowed on me and my Performance, and for his Arguments and Objections againſt it, as he calls them, that I think they are already anſwered or obviated, as far as the Matter of them deſerves and requires. All I ſhall add is, that I would recommended it to him to look into his own firſt Treatiſe on this Subject, if he thinks it worth his Trouble, which few perhaps beſides himſelf will do, and compare what he ſays in P. 27 and 28. with what he ſays in P. 83, *&c.* In P. 27, having made a Suppoſition that three Perſons are to enjoy an Eſtate by equal Portions during their joint Lives, Re-

mainder to the Survivors and Survivor, he makes a Quære what is the Value of each Man's Right; and, in the next Page, ſolves it thus—*That each Perſon has a Right to one Third of the Value of their joint Lives, and to Half the Value of the Expectation after the Deceaſe of either of the other two, and alſo to the whole Value of the Expectation of both the others; which three Sums added together give the Value for each.*—In P. 83, *&c.* he ſuppoſes a Man to give his three Daughters an Annuity of 20*l. per Annum* each for their Lives; and to order, that, when any of them dies, the Annuity of the Siſter dying ſhoud fall to the other two in equal Shares; and, at the Death of another, that the whole ſhoud belong to the Survivor for her Life. He goes on then to compute the Value of this Gift to each of theſe Women; and gives us the Value of the Share of the eldeſt Siſter thus: —Firſt, 20*l. per Annum* during her own Life. Secondly, 10*l. per Annum* in Reverſion of the youngeſt, to continue from thence during the joint Lives of the other two. Thirdly, 10*l. per Annum* in Reverſion of the Second, to continue from thence during the joint Lives of the Firſt and Third. Fourthly, 40*l. per Annum* in Reverſion of the two youngeſt Lives during her own Life. Now, unleſs this Writer does, and until he has reconciled his Solution in the firſt Paſſage, in which he allots to each Perſon three Sums only, with his Computation in the ſecond Paſſage, where he brings to Account four Sums: Which I ſuppoſe he will be in no great Haſte to attempt,

and,

and, if he does, will never be able to effect: Som People will, perhaps, call in Queſtion his Faculties, Algebraical, Mathematical, or Intellectual, and, for my own Part, I ſhall never ſay a Word more to ſuch a ——.

To Mr. *Hodgſon*, who ſtiles himſelf *F. R. S.* and Maſter of the Royal Mathematical School, who has lately publiſhed ſom Operations and Tables on the Value of Annuities for Lives, I have ſomthing more to ſay, becauſe he builds, in Part at leaſt, on the ſame Foundation on which Dr. *Halley* did, and has revived, ſeemingly, and unneceſſarily I think at this Time of Day, the Controverſy on the Rule for making the Valuation of theſe Annuities. In his Preface, by Way of Apology for framing a new Table of Mortality for *London*, to ſerve the Place of the old one of *Breſlaw*; he tells us, that at *Breſlaw* one Half of the People that are born there live 'till they are about 41 Years of Age, in *London* that one Half die before they are of the Age of 10 Years. Where and whence this Writer pickd up this Notion about the Age to which the People who are born at *Breſlaw* do live, or who ſhould tell him ſo, I cannot deviſe; for Dr. *Halley*, in his Treatiſe on this Subject, tells us, in ſo many Words, that it appears from the *Breſlaw* Table, and in Fact it does ſo appear, that one Half of thoſe who are born there are dead in 17 Years: But, this not being material to the main Deſign, let it paſs. In his Book, after giving us, for four or five Pages, a Calculation of the Chance of the Duration of

Lives

Lives for *London*; tho' by the Way, this long Detail, even were it a good one, is very little to the Purpoſe, unleſs ſuch Chance be the Meaſure of the Value of the Annuities depending on thoſe Lives; he ſays, that from thence the Value of an Annuity for a ſingle Life of a given Age, or the Value of any limited Time the Perſon has a Probability of living, at any Rate of Intereſt, may be found. He proceeds, by Way of Inſtance of a Perſon of 10 Years Age, and at 3 *l. per Cent.* Intereſt, to compute the Value of an Annuity for his Life of 1 *l. per Annum*; and having told us the preſent Value of 1 *l.* payable at the End of one Year, when 'tis an abſolute Certainty, and computed what muſt be deducted thereout on the Life Annuity for that Year, on Account of the Chance of Mortality ariſing in that Year; and having gone through the like Operation on the ſecond and the third Year of the Life of ſuch Perſon; he concludes, that, *by repeating theſe Operations to the utmoſt Extent of poſſible Life, the Value of* 1 l. per Annum, *for a Life of* 10 *Years old, will be found to be* 20-16-46, *or* 20-1646 *Years Purchaſe:* And then adds, that *the Tables there following were conſtructed after this Manner.* I obſerve here, that this Method of computing the Value of Annuities on a ſingle Life, tho' it may vary in the Expreſſion and Shape of it, yet in Subſtance is the ſame as Dr. *Halley* preſcribed for that Purpoſe; and therefore, that all the Exceptions taken to this Rule, as Dr. *Halley*'s, will equally lie againſt it now adopted and abetted

by

by this Author. This being premiſed, I ſay, that this Writer has either himſelf been impoſed on, or attempts to impoſe upon his Reader, when he tells him that the Tables there following were conſtructed in the Manner mentioned; for, whatever was the Manner in which they were conſtructed, 'tis certain it was not this; and he himſelf ſhows it demonſtratively, and ſo plainly and palpably, that one may read it as he runs. He ſays, on computing the Value of an Annuity for a Life of 10 Years Age, the Value or Years Purchaſe appears to be, when computed by his Method, 20-1646; which I believe to be right, becauſe I have made the Computation on the ſame Life, at the like Intereſt, and by his Rule, and I make the Value 20-1623; and this very ſmall Difference may poſſibly ariſe by his inſerting an Account of the Fractions at the End of each Year, and my omitting them. But in the Table, which follows in the very next Page, he gives 19-5761 as the Value of an Annuity for a Life of 10 Years of Age, Intereſt computed at the Rate of 3*l. per Cent.* when, in the Paragraph juſt preceding, he had computed and ſtated it, and at that Rate of Intereſt, at 20-1646. Now the Sum 20-1646 is equal to a Term of 31 $\frac{1}{8}$, and the Sum 19-5761 only to a Term of 29 $\frac{7}{8}$; ſo that there is the Difference of a Year and an Half in the Term to which one and the ſame Life is equal, as computed in the precedent Paragraph and as given in the ſubſequent Table. At the Rate of 5*l. per Cent.* there is the like Variation, but on the

the contrary Part; for the Value, on computing by the Rule given by this Writer, is only 15-0430, which is equal to a Term of 28 ½, but in the Tables is put down 15-3588, which is equal to a Term of 29 ⅜. At the Rate of 4 *l. per Cent.* indeed, the Value ariſing on a Computation by the Rule and that inſerted in the Tables is one and the ſame, *viz.* 17-2738, which is equal to a Term of 29 ⅜. From theſe Calculations and Facts, 'tis evident that the Value of this Annuity, at the Rate of 3 *l.* and 5 *l. per Cent.* was not collected and aſcertained on a Computation by the Rule, for they are not agreeable to it; but, on the Contrary, it muſt have been firſt ſuppoſed that the Values at the Rates of 3 *l.* and 5 *l.* were equal to the ſame Term to which the Value at 4 *l.* is equal, and then a Value correſponding to that Term is inſerted as the Value of the Annuity at the Rates of 3 *l.* and 5 *l.* The Truth is, and in his Table, P. 40. and which he calls, *The Value of Lives upon Annuities certain*, 'tis manifeſt, that this Writer did mean and intend that the Value of Annuities for a Life of the Age of 10, and indeed of all other Ages of Life, at all Rates of Intereſt, ſhoud reſpectively correſpond to one and the ſame Term in an Annuity certain; for this Table exhibits them all, at every Rate of Intereſt, as equal to one and the ſame Value in an Annuity certain, which is the ſame Thing as ſaying, that they are all equal to one and the ſame Term in an Annuity certain: And that is the very Point for which I contend, and ought

to

to be, but is not preſerved, nor can poſſibly be obtained by Dr. *Halley*'s Rule, and in the *Eſſay* have demonſtrated it, tho' by this Author's new-fangled Regulator it may.

Now this Method of conſtructing the Tables of theſe Values does not only contradict the Manner by which he ſays, in the precedent Page, he conſtructed thoſe Tables, but 'tis likewiſe an abſolute and total Departure from the Rule laid down by Dr. *Halley*, obſerved by others, and pretended to be aſſumed by this Author, except in the ſingle Article of 4 *l. per Cent.* and is conformable to the Rule laid down in the *Eſſay*, ſo far forth as it makes the Term in an Annuity on one and the ſame Life to be the ſame as the Term in an Annuity certain, at all Rates of Intereſt. He does indeed by this Means, in Part, obviate or evade the main Objection made to Dr. *Halley*'s Rule, but we ſhall ſee preſently that my Exception to this Method remains as ſtrong, at the Rate of 4 *l. per Cent.* as it does againſt the old Rule, where the Rate of 6 *l. per Cent.* is uſed. The Exception I mean is, that, by Virtue and in Perſuance of the old Rule, ſuch a Sum is produced as the Value of a Life Annuity, as that one and ſame Life ſhall be equal to a different Term as the Rate of Intereſt varies; and it is demonſtrated that this muſt be the Caſe, and admitted ſo to be by Mr. *H. B.* whom I have beforementiond; and conſequently that the Duration of a Life or the Chance of it depends on the Rate of Intereſt, and muſt, and in Fact is found to vary as oft as that

varies. Since then the Term in the Value of an Annuity at every other Rate of Intereſt different from it is by this Method to be aſſized and brought to the Standard of the Term in the Value of an Annuity at the Rate of 4 *l.* then the Chance of Duration of Lives depends on that particular Rate of Intereſt as the common Standard. Now if the Queſtion were asked, on what Pretence or by what Authority this particular Rate of Intereſt is made the Regulator here, I think no tolerable Anſwer coud be given but this, that at preſent this is the common Price of Money. Be it ſo: But what if it ſhoud ſo happen that in 20 or in 50 Years Time, more or leſs, the Price ſhoud alter, and ſink to three, or riſe again to five? Why, we muſt then change our Standard as the Caſe ſhall come out; and whereas $29\frac{7}{8}$ on a Life of ten Years Age is now the Term to which an Annuity on ſuch a Life is equal, then *in futuro*, if the Price ſhoud be three, an Annuity on ſuch a Life woud be equal to a Term of $31\frac{3}{8}$, and, if it ſhoud riſe to five, woud be equal to a Term of $28\frac{1}{2}$. If we look backwards, we find the Price of Money about 50 or 60 Years ago to have been ſix, and about an hundred Years ago eight; and in the firſt Caſe the Term in the Annuity for the like Life woud be $27\frac{1}{4}$, and in the latter about 25, and then theſe Terms reſpectively muſt be lookd upon as the Term to which an Annuity on ſuch a Life, at the ſeveral Times, and at the different Rates of Intereſt then courant, muſt be reduced as to a common Standard. Now this Alteration of the

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common Standard changes the Chance of the Duration of a Life of one and the ſame Age, in the ſame Manner and as effectually as the Change of the Rate of Intereſt, at any one Time, does change the Chance of the Duration of one and the ſame Life at that Time: And I believe no Body will be ſo hardy as to aſſert, that the Chance of the Duration of a Life of one and the ſame Age is not the ſame now as it was 50 or 100 Years ago, or that it will not continue the ſame for 50 or an 100 Years to come; ſo that the Difference between the Dr's Rule and this Regulator woud be this and no more than this, that by the Uſe of the former the Chance of the Duration of one and the ſame individual Life is varied as the Intereſt varies, and the Life is made ſhorter or longer, as that is higher or lower; and by the Uſe of the latter, that ſuch Chance woud vary on a Life of one and the ſame Age, in the ſame Manner and Proportion; from whence it follows, that this new Regulator is liable to the very ſame Objection as the old Rule.

This Writer can himſelf give the beſt Account, what induced him in his Book to recommend and make an Operation by the Rule, but in conſtructing his Tables to give it up and deſert it, and to fall into this new invented Method; but 'tis ſuſpectable that he ſaw and felt the Weight of the Exception to the Rule, and not being able to anſwer it, as I may be confident he was not, he enterd into this Practice to elude or to cover it. But, whatever his Reaſon might be for uſing theſe Ar-

tifices, he has faild of Succeſs; as we ſhall ſee preſently he has done likewiſe in putting in Practice the ſame little Arts, with Reſpect to another material Exception to the Rule mentiond in the *Eſſay*. The Exception I mean here is; that in uſing the Rule preſcribed, in the younger Life, the Term, to which the Chance of the Duration of the Life is equal, is longer than the Term to which the Value of the Annuity for the Life is equal: And on the Reverſe, in older Life, the Term, to which the Value of the Annuity for the Life is equal, is longer than the Term to which the Chance of the Duration of the Life is equal: And this in the *Eſſay* was ſaid to be contradictory to common Senſe and Reaſon; and notwithſtanding the learned Endeavors of Mr. *H. B.* in his Obſervations before-mentiond, to reconcile theſe Inconſiſtencies, I ſtill adhere to my former Opinion, and am confirmd in it, on ſeeing the great Labor he has taken, and the little Succeſs he has met with, in the Attempt. Let us examin now how this Matter ſtands on computing the Value of theſe Annuities by this new Method. In the Caſe of the Value of an Annuity for a younger Life, ſuppoſe of ten Years Age, 'tis equal to a Term of $29\frac{7}{8}$; and the even Chance of the Duration of ſuch a Life is 35 Years, Computation in both Inſtances being made by this Writer's Tables, ſo that the Difference amounts to a ſeventh Part of the whole Term, and ſomething over: But in the Caſe of an Annuity for the older Life, ſuppoſe of 80 Years of Age, 'tis ſet down as equal to a Term of

of 4-9103, which is four Years and eleven Months, wanting about three Days; and if we inquire, to what Term ſuch a Life on the even Chance is equal, this Writer *p.* the *9th* ſays between 4 and 5 Years; in *p.* the *7th* five Years; and, if we compute ſuch Chance by his Bills of Vitality and Mortality for *London*, I find, and if any one elſe will give themſelves the Trouble of computing it, as I have done, he will find, that the even Chance is juſt four Years and ten Months; ſo that the Duration of the Annuity exceeds the Chance of the Duration of the Life, as well in Computations by this new Method, as by the old Rule; and therefore both of them abſurd and defenceleſs.

After all, what do theſe little Subterfuges ſignify? Even juſt nothing at all; for Mr. *H. B.* a ſtrenuous Advocate for Dr. *Halley*'s Rule, has admitted the Fact to be ſuch as I have ſtated it on an Operation made by ſuch his Rule, and, on the like Operation by this Writer's Rule, the Fact muſt come out nearly the ſame, and conſequently my Exception to the old Rule and this new Method will be equally ſtrong, tho' perhaps it may not appear in a Light ſo clear, even tho' the Chance of the Duration of this Life were ſuppoſed to be full five Years, and to exceed the Term in the Annuity, or that the Terms in both were equal. The Way of Reaſoning here ſtands thus; Since by Virtue of the Rule the Term in an Annuity for a Life is diminiſhed and made leſs than the Term to which the Life on an even Chance is equal, 'tis neceſſary that a certain and regular

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Proportion of ſuch Decreaſe ſhoud be preſerved through all the Stages of Life; but here, where in younger Life the Term in the Life is 35 Years, and the Term in the Annuity is 29 $\frac{7}{1}$, and in the elder the Term in the Life is 5 Years (ſuppoſing it were ſo) and the Term in the Annuity four Years and eleven Months, or very near it; the Decreaſe in one Caſe is one Year in ſeven, and in the other one Month only in 55 Months.—And if we ſhoud ſuppoſe that in ſom Inſtance the Term in the Chance of the Life was the ſame as the Term in the Annuity, which muſt be the Caſe in ſom one Inſtance, tho' not here: And if it ſhoud be ſaid that there the Value of the Annuity woud be the right one; I will admit it to be ſo, and admit it for that Reaſon, becauſe it has fortuitouſly happend in that Caſe, that the Term in the Life and the Term in the Value are the ſame and coincide. But let us look a little into the State of an Annuity on a Life ſtill older, ſuppoſe of 90 Years Age, and ſee how Matters ſtand there. The even Chance of ſuch Life, *p.* the 9*th*, is ſaid to be a Term of three Years, and the Value of an Annuity for ſuch Life given in his Tables is 1-8772; but this is not near equal to a Term even of two Years, for a Term of two Years is in Value 1-9135, ſo that the Difference between them will exceed a whole Year; and conſequently the Decreaſe in this Caſe is one third at leaſt of the Term, and this is a greater Decreaſe than in a Life of ten Years Age, for there the Decreaſe is only a ſeventh Part of the Term. We find then that to

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the old Difficulties on the Rule an Addition is here made of another; and, unleſs Mr. *Hodgſon* can diſcover a better Solution of them than *H. B.* has done, theſe Irregularities in the Proportions of the Decreaſes, believe me, do effectually deſtroy the Juſtice of the Regulator.

In the *Eſſay* very little Notice is taken of Mr. *Hayes*'s Method of Valuing Annuities on Lives, becauſe his Tables drawn for that Purpoſe ſeemd to me ſo abſurd and wide of the Mark, that I coud not not think that any one, of tolerable Underſtanding in Things of this Nature, woud be guided by them: But ſince his Treatiſe on this Subject has lately received a new Impreſſion, in a very ſuperb Dreſs, I muſt ſay ſomething to it now. I obſerve he gives us the Age of 30 as the firſt Age of Life, on which he begins his Valuation of Annuities; but for what Reaſon he omits all the Stages of Life precedent to that of thirty I cannot imagin, unleſs he were of Opinion that the Age of thirty was the beſt Age of Life: But for this Surmiſe there's no ſort of Foundation in Nature or Experience; and on the contrary the *Breſlaw* Table, the Bills of Mortality for *London*, and thoſe of the Nominees in the Government Annuities on Survivorſhip, which is the beſt Guide of all, do plainly ſhow, that 'tis quite a vulgar Error. In *p.* the *1ſt* of his Book, as the Foundation of his ſubſequent Calculations, he tells us, *that the Annuities, calculated in the following Pages, ſhow, at one View, the preſent Value upon the Life of any Age from* 30 *to* 73, *according to the Chance*

Chance of an Annuitant's living to the Extremity of the common oldeſt Age of Life, and this is done upon the Suppoſition of the various Degrees of Probability, which Lives of different Ages have to continue in Being. In *p.* the 22*d*, we have a Table, ſhewing how long an Annuitant muſt live to receive the Value of an 100 *l.* ſunk, ſuppoſed to be given for his Annuity, according to the Chances of Life, at the ſeveral Rates of 4, 5, 6, 7 and 8 *per Cent.* Intereſt: And, in *p.* the 74*th* to 127*th*, we have a Table, a very long Table, for the Value of an Annuity upon a Life from the Age of 30 to 73, at the like Rates of Intereſt, and theſe I believe, do very well agree and coincide with each other. But then theſe two Tables, if right, do manifeſtly demonſtrate, that this Writer's firſt and fundamental Poſition is abſolutely falſe; or the Poſition, if that be right, as undoubtedly it is, demonſtrates that the Tables are all falſe, for theſe Tables coud not be framed, and 'tis viſible they were not framed, according to, or with any Regard had to the Principle there laid down, *viz.* the Chance or Probability of the Life of an Annuitant, for there can be no Chance or Probability of that happening in Life, which is impoſſible ever to happen. I confeſs it exceeds my Comprehenſion, by what Means or on what Grounds this Author coud be induced to ſuppoſe, even in the firſt Inſtant, and flatly in Contradiction to his own firſt and fundamental Poſition, that the Chance or Probability of a Life of one and the ſame Perſon coud by any Ways or Means whatever

ever be varied; and yet his Tables here are founded upon a Suppoſition, that ſuch Chance does vary as the Rate of Intereſt varies, for his firſt Table gives us on a Life of 30 Years Age the Chance or Probability of his living for 25 Years at 4 *per Cent.* Intereſt, and at 8 *per Cent.* for 16 Years only; and the Inſtance following ſhows moſt plainly that this is intirely falſe.

From his Table in P. 22, *&c.* which ſhows how long an Annuitant muſt live to receive the Value of the Money he gave for his Annuity, we find that if he buys at 4*l. per Cent.* Intereſt he muſt live 25 Years, and if he buys at 8*l. per Cent.* Intereſt he muſt live 16 Years. Now if one and the ſame Perſon buys two Annuities for his own Life, and purchaſes one at the Rate of 4*l. per Cent.* Intereſt, and the other at the Rate of 8*l. per Cent.* then ſuch Annuitant, in order to receive the Value of his Principal Money on one of the Annuities, muſt live 25 Years, and, with Regard to the other, he need to live only 16 Years: But 'tis in Nature impoſſible that his Life ſhoud continue for 25 Years, if it ends in 16 Years; and almoſt a Contradiction in Terms, to ſay, that the ſame Annuitant has a Chance or Probability that he ſhall live 25 Years, and at the ſame Time a Chance or Probability that he ſhall live only 16 Years. Beſides, theſe two Bargains can't poſſibly be fair and juſt ones on both Sides, for if the Annuitant has a Chance or Probability to live 25 Years, he has over-reachd the Vender who ſold at 8*l. per Cent.* becauſe with Regard to

him the Annuitant will have been reimburſed the Principal, or received the Value, in 16 Years; and, on the other Side, if the Annuitant has a Chance to live only 16 Years, then with Regard to the Grantor at 4*l. per Cent.* the Annuitant himſelf will have been cheated, becauſe there will be nine Years wanting to make up his Principal, or to receive the Value of the Money given on the Purchaſe. In ſhort, if this Writer's fundamental Poſition, *viz.* that the Chance or Probability of the Continuance of a Life be the Meaſure of the Value of an Annuity for the Life, be a right one, as I make no Doubt it is; or indeed if he intended to make Uſe of ſuch Chance as his Rule for conſtructing his Tables; inſtead of his long Rows of Numbers and Figures, which are extended for 50 Pages together and upwards, he might even as well have ſet out ſo many broken Reeds, for he that makes Uſe of one as his Guide, in the Valuation of Life Annuities, will be led into groſs and very miſchievous Errors, as ſurely as he will be deceived in his walking who leans on ſuch rotten Staffs for his Support. This Writer, I think, ſomewhere in his Treatiſe, inſinuates, that he grounded his Calculations and Rules on the Practices of ſom of the great Companies in *London*, or, *vice verſâ*, that they grounded their Practice on his Rule; but, whichſoever of the two were the Caſe, I believe moſt People at preſent will be ready to pronounce, that ſom Events have ſufficiently damned both the Rules and the Practice.

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I ſhall not need to add any Thing farther on this Subject, unleſs ſomewhat to inform the Reader on what Grounds I framed the Tables for the Value of an Annuity on a ſingle Life, and in what Manner they were conſtructed; ſo that he may be able to judge for himſelf how far he may depend on them. I adhere then to the Poſition laid down in the *Eſſay*, that the Chance of the Duration of the Life is the only ſure Meaſure of the Value of an Annuity depending on that Life; for no material Objection to it has occurred to me which has not been obviated or ſufficiently removed. The Chance of Vitality being then the Ground-work, in order to inveſtigate this, I reviewed the Bills of Mortality for *London*; and having an Opportunity of ſeeing and conſidering an Account of the Mortality of the Nominees in the Government Annuities on Survivorſhip, I had ſpecial Regard to ſuch Account; ſince it containd a Bill of Mortality of Individuals without Variation, and therefore much the beſt Guide to diſcover and fix the Chance of Vitality.—

Now, ſince there is one certain Stage of Life which muſt have the greateſt Chance of Vitality; for thoſe Chances, from the Beginning of Life, do increaſe to a certain Time, and from thence decreaſe to the End of Life; my firſt Inquiry was what might be that Year or Years of Life wherein the Chance of Vitality was greateſt.—To fix this Point, I conſulted firſt the *London* Bills of Mortality, and thoſe of the Nominees in the Government Annuities, and from theſe fixed the Age

between ſix and ten, including both, as the beſt Stage of Life, on the following Grounds. On a View of thoſe *London* Bills, it appears that the Number of Perſons dying between 10 and 20 Years Age, is 30 in a 1000; and in the Period between 5 and 10 Years age, tho' no more than Half the Length of the former Period, the Number is 38 in 1000; and in theſe two Periods the Proportion of Mortality is the loweſt: But then, tho' the Proportion of Mortality in theſe two Periods is not the ſame, and in the Period of younger Life is much the greater, I conſiderd that the Children of the Ages between five and ten were reſpectively and gradually removed farther from the Extremity of Life, and that the one might nearly counterbalance the other, I ſtated all thoſe five Years at one and the ſame Chance of 35 Years. I then examind how the Caſe ſtood in the Account of the Mortality of the Nominees in the Government Annuities, and found it to ſtand thus: The Perſons were nominated ſome time before *Midſummer* 1693; and at *Chriſtmas* 1693, the Account I have ſeen being drawn up from *Chriſtmas* to *Chriſtmas*, the Number of Perſons then ſurviving was 1009, and at *Chriſtmas* 1728 there were dead 505; ſo that in 35 Years one Half exactly were dead; and I ſuppoſe that all the Perſons nominated, and when nominated, at an Average, might be ſomething between the Age of ſix and ten. To ſatisfy myſelf farther in this Point, I made a Computation on the Bills of Mortality for *London*, and obſerved the

the ſame Method preſcribed by Dr. *Halley*, and made Uſe of by him and Mr. *Hodgſon*, and another on the ſame Bills in a Method formd by myſelf, and the Produce for the even Chance of Vitality for a Life of ten Years Age is, upon all the Operations, very nearly the ſame; and, in ſom of the Ages of Life younger than ten, this Chance runs rather higher.—

Having thus ſtated the Chances of Vitality for the beſt of Age of Life, *viz.* of ſix to ten, and ſuppoſed it to be equal to a Term of 35 Years; and ſince an Annuity for ſuch Lives will be equal to the ſame Term; moſt People, I believe, will think the Eſtimate ſufficiently high, even for that very beſt Age. From thence I proceeded to the Stages of Life precedent and ſubſequent to this, and propoſed to ſettle the Chances of theſe by inquiring into and fixing the Proportions of the Decreaſes of them from the ſaid Term of 35 Years, for every ſingle Year, or for every Period of ten Years, reſpectively. Here I was in Hopes of ſaving myſelf ſom Trouble, by uſing the Labors of ſom of my Predeceſſors who have wrote on this Subject, and particularly of Mr. *Hodgſon*, who in his Treatiſe, P. 6, has given us a Computation of theſe Chances, from every Period of ten Years of Life, from ten Years Age to the Extremity of Life. For this Purpoſe, I threw ſuch his Computation into a ſhort Sketch, exhibited at the End, in Scheme No. 1. but, on doing this, I diſcovered immediately that ſuch Scheme did not give us a right Partition of thoſe

thoſe 35 Years or Chances for all the ſeveral Periods of ten Years these mentiond. The Defect in it appears almoſt on the Face of the Scheme; but, in the Inſtance which follows, 'tis ſo groſs, that it cannot fail of being ſeen, when pointed out, by every Man of common Underſtanding. That the Chance of Vitality on a Life for one Year does ſink in that Proportion in which a certain Number of Perſons living at the Beginning of ſuch Year does die before the End of it, is a plain and an indubitable Truth: And yet the Decreaſe of theſe Chances given in the Scheme, in ſom Periods of ten Years, where we throw ſuch ſingle Years into a Period of ten Years, runs counter to that Poſition, directly, and to a very wide Degree. For Inſtance, in the Period from ten to twenty Years Age, the Chance of Vitality is ſunk eight Years, and conſequently in Proportion for each ſingle Year of theſe ten Years, and in the ſucceeding Period, 'tis ſunk only five Years and an Half; and yet the Perſons dying in the firſt Period are only 31 out of 490 living at the Beginning of it, and in the ſecond the Perſons dying are 94 out of 459 living; from whence it appears that the Number of Perſons dying in the ſecond Period is much more than double the Number of thoſe dying in the firſt, and yet the Chances of Vitality ſunk in ſuch ſecond Period are not near three Quarters of thoſe ſunk in the firſt: Therefore the Partition in this Scheme, not giving the true Decreaſes of theſe Chances of Vitality for ſom Periods and Years of

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Life, cannot give us a juſt Partition of theſe Chances for the whole Life. From thence it will follow, that the Tables for the Value of Annuities for a Life, conſtructed by Mr. *Hodgſon*, do not give us the true Value of them, even tho' they were in all other Inſtances faultleſs; for that the Value muſt depend on ſuch Chances, and he himſelf ſuppoſes as much, and yet theſe Chances themſelves, in ſom Parts of Life, are not rightly adjuſted, by this Method; and this is the Caſe in all the Stages of Life from the 10th to the 60th Age of Life, or near it; and, believe me, this is a very large, and, in all Reſpects, a conſiderable Part of Life.—

I cannot forbear adding a few more Words on this Subject, becauſe I think that, from this Writer's own Poſition, it may be demonſtrated, that his Repartition of the Chances of Vitality, at leaſt for a Life between 10 and 20, muſt be wide of the Mark. In his making the Computation of theſe Chances, I ſuppoſe that he made Uſe of his own Tables of Vitality for *London*, by computing in what Time the Number of Perſons living of the Age of 10, of 20, and 30, were reſpectively reduced to an Half, and then taking ſuch Time as the Term to which Perſons of thoſe reſpective Ages were equal; for that, on ſuch Computation, and on ſueh Operation, they do come out to be the ſame as we find them in his Treatiſe, *viz.* 35, 27, and 21 ½. Now as the Decreaſes of the Chances between 10 and 20, and between 20 and 30, appeard to be unproportionate,

tionate, the firſt being eight Years by the Death of only 31 Perſons out of 490, and the ſecond being no more than five and an Half by the Death of 94 out of 459; I was ſo much ſurprized, as to miſtruſt this Method ſo far, as to examin it more accurately than I had done when I wrote my *Eſſay*, for there I gave into and made Uſe of it; but, on a cloſer Examination and frequent Trial of it, I am ſatisfied that the Rule will not hold in all Inſtances, tho' in ſom it does. I lookd then into the Poſition on which this Writer grounds and makes the Calculation of the Chances which immediately there follows; and, on a Calculation made by the Method preſcribed by that preliminary Poſition, it appears, that 'tis 14, and much above, to 1, that a Perſon of 10 Years old lives to the End of the ſucceeding Period of 10 Years; and that 'tis only 5, and a little above, to 1, that a Perſon of 20 Years lives to the End of the ſucceeding Period of ten Years: From whence it follows, that the Chances of Vitality for the Lives in the Period between 10 and 20 Years, is very far from being rightly apportiond; —for he who is 10 Years of Age, and has the Odds of 14 to 1 to live 10 Years, has evidently a greater Chance of living to be 20 Years old, than he that is 20 Years of Age, and has only the Odds of 5 to 1 to live 10 Years, has of living to be 30 Years old: And yet this Author's Computation of the Chances of Vitality of a Perſon of 10 Years, and a Perſon of 20 Years Age, makes theſe Chances of the Perſon of the younger Life

to

be greatly leſs than the ſame Chances on the older Life. This being the Caſe, on the Method made Uſe of by this Author for adjuſting the Chances of Vitality, I conſulted Dr. *Halley* on this Subject, but I find that he preſcribes and makes Uſe of the ſame Method as this Author does; but I muſt beg Leave, in this as well as in ſom other Matters of this Kind, to diſſent from them both, and to affirm that this Rule does not in all Inſtances, tho' it may do in ſom, give us the even Chances of Vitality, and that for the Reaſon and on the Computations juſt now made; and thoſe are built upon the Foundations laid down by Dr. *Halley* himſelf, where he gives the Rule to find the Odds, which a Perſon of a given Age has to live for a given Term. Being forced thus to reſort to a different Method, I formd that which follows, both to aſcertain the even Chance of Vitality for a Life, and for any Period of Years: And this differs very little from that given by the Dr. for calculating the Odds of Vitality for a certain Number of Years, in another Shape indeed, and with ſom Additions.

The Rule then is this: To take the Number of Perſons living at the Beginning of any Year or Term of Years, and the Number living at the end of that Year or Term Years; and if we ſubtract the one from the other, the *Remanet* is the Number of Perſons dying in that Year or Term of Years: Next, to take the Number of Years, which we may call Chances of Vitality, to which a Perſon of the Age named may live, if we compute to the Ex-

tremity of Life: and then to inveſtigate what is the Proportion in ſuch Years or Chances of Vitality, which the Number of Perſons living at the Beginning of ſuch Year or Term of Years bears to the Number of thoſe dead at the End of the Year or Term of Years. To perform this, we muſt multiply the Number dead in the Year or Term of Years by the Number of Years or Chances of Vitality through the whole poſſible Life, and divide the Produce by the Number living at the Beginning of the Year or Term of Years: And the Quotient will give us the Chances of Vitality ſunk in that Year or Term of Years: And, if we deduct the Number of Chances ſunk in that Time out of the Number of Chances ſubſiſting at the Beginning, the *Remanet* will be the Number of Chances due and belonging to the Life propoſed, and the Term to which ſuch Life will have an even Chance of Duration.

Or thus: In Caſe the Number of Years or Chances of Vitality for any Age of Life be ſtated and known; if we want to know what be the Number of Years or Chances of Vitality due and belonging to a Perſon of the next or any other ſubſequent Age of Life, we need only to take that Number of Years or Chances of Vitality, to which the Life of the Perſon of the Age next precedent to the Age of the Perſon after whoſe Chances of Vitality we are inquiring; and then to inveſtigate what is the Proportion in ſuch Years or Chances of Vitality, which the Number of Perſons living at the Beginning of ſuch Year or Term

of

of Years bears to the Number of thoſe dead at the End of the Year or Term of Years, and then to proceed as before directed.

I ſhoud obſerve here, whether we take a Number of Years or Chances, according to all the poſſible Years or Chances ſubſiſting on the Life given, which in a whole Life may be ſuppoſed to be an 100, it being poſſible that a Perſon juſt born may live to be an 100 Years old; or whether we take ſuch a Number of Years or Chances, as anſwers to the even Chance of Vitality on the Life of a Perſon of the Age next precedent to the Age of the Perſon of whom the Inquiry is made, where ſuch even Chance is already ſtated and known, the Decreaſe of the Years or Chances in each Life or Term of Years muſt be and will be exactly in the ſame Proportion. So that either of thoſe two Numbers may be uſed with equal Succeſs.

As this Way of Reaſoning and the Rule may ſeem a little abſtruſe, I will apply it in an Inſtance or two on the Perſons named in the Government Annuities on Survivorſhip. The Nominees in theſe Annuities were named at or before *Midſummer* 1693, and at *Chriſtmas* 1693, when my Account begins, the Number of them then living was 1009, and I did ſuppoſe that the Age of each Nominee, at an Average, might be about nine or ten. We want to know then to what Age any one of theſe Nominees might probably live, or what were the Chances of Vitality due and belonging to any one of them, ſuppoſing him

to be ten Years old, and as of ſuch Age to have 90 Years or Chances of poſſible Life. The Number of Perſons living at the Beginning of this Year, *viz.* at *Chriſtmas* 1693, was 1009: At the End of the Year 1728, there were living 504, and, if we deduct 504 out of 1009, the Number remaining will be 505, that is, in that Compaſs of Time died 505. The Age to which any one of theſe Nominees might poſſibly live, being all ſuppoſed to be ten Years old, is 90. Then under the Direction of the Rule we multiply 505, the Number of the dead, by 90, which is the Number of Years or Chances through the whole poſſible, and the Produce is 45450: And if we divide this Produce by 1009, which is the Number of Perſons living at the Beginning of the Term, the Quotient comes out to be 45, a ſmall Matter over. From hence, we find that between *Chriſtmas* 1693 and *Chriſtmas* 1728, that is, in 35 Years or very near it, one Half of theſe 90 poſſible Chances were loſt, and conſequently the other Half left: Therefore the Life of one of theſe Nominees, on the even Chance, was equal to a Term of 35 Years, or might probably continue in Being for ſuch a Term; and on this Computation, and ſom others before mentioned, I ſtated the even Chance of a Life of ten Years Age in general at 35 Years, as I found it to be in this Particular.

To give another Inſtance: We will compute the Chances of a Life, where we will ſuppoſe that thoſe Chances of Vitality for a Life of ten Years

Age

Age is fixed and known; and there will make Uſe of the ſecond Part of the Direction of the Rule. The Number of Nominees in the Annuities on Survivorſhip living at *Chriſtmas* 1693 was 1009: The Number dying between *Chriſtmas* 1693 and *Chriſtmas* 1703 was 117. The Age of each Nominee, at an Average, we have ſuppoſed to be ten Years or ſomething near it: A Life of ten Years Age, on the even Chance, we have ſtated as equal to 35 Years: Now, if we multiply 117 by 35, the Produce will be 4095, and, if we divide that Number by a 1009, the Quotient will be 4, and a little over: So that the Decreaſe of the 35 Years or Chances of Vitality in thoſe ten Years will be 4 Years and a ſmall Matter over; and, if we deduct 4 out of 35, the Years or Chances of Vitality remaining will be 31, and this will be the Number of Years or Chances for a Life of 20 Years Age. If we proceed in like manner on a Life of 30 Years Age: The Number of Perſons living at *Chriſtmas* 1703, ſuppoſed then to be of the Age of 20 or near it, was 892: A Life of 20 Yeas Age, on the even Chances, has been computed and ſtated as equal to a Term of 31 Years: The Number of Perſons dying between *Chriſtmas* 1703 and *Chriſtmas* 1713 was 169: And, on the like Operations as before, the Decreaſe of the Years or Chances in that Time will be 5 Years and $\frac{7}{8}$, or near ſix Years; and, if we deduct ſix Years out of 31, the Years or Chances then remaining will be 25,

and

and will be the Number of Years or Chances for a Life of 30 Years Age.

By this Rule, and in this Manner I computed the Decreaſe of theſe 35 Years or Chances, by Periods of ten Years, from the Age of 10 to an 100; and made Uſe of the Bills of Mortality, in the Annuities on Survivorſhip from the 10th to the 68th Year of Life, which was as far as thoſe Bills coud conduct me, and from thence, being deſtitute of this ſure Guide, for the four Periods of remaining Life, I made Uſe of the *London* Bills of Mortality, obſerving ſtill my own Rule: And by theſe Decreaſes I conſtructed the Value of Annuities exhibited in the Table annexd. I have before computed, on the Bills of Mortality for *London*, the Decreaſe of theſe 35 Years or Chances of Vitality, for each Period of ten Years, from 10 to an 100 Years Age, from the Account given of them by Mr. *Hodgſon*, and have ſet out the Proportions of them for each Period of ten Years in Scheme N. I. — Here I have made a Computation likewiſe, in my own Method, of the Decreaſes of the ſame Chances on the ſame Bills of Mortality, for the ſame Periods of Life, and the Proportions of them, as ſtated by me, ſtand as in Scheme N. II; and I believe it may be ſafely left to the Reader, on a near Inſpection on thoſe two Schemes to diſcern which is neareſt a regular and proportionate Decreaſe, and deſerves the Preference; eſpecially if this Conſideration be taken along with it, *viz.* that

that this Writer does in expreſs Terms ſay, that, *after Perſons arrive at ten Years, the Decrements of Life are little in Compariſon with the former Years, and decreaſe regularly.*

I aſſert then, on computing by my Rule, and on the *London* Bills of Mortality, that the Years or Chances of Vitality loſt in the ten Years, between the Age of ten and twenty, are no more than $2\text{-}\frac{7}{8}$, as ſtated in the Scheme, N. II; and that a Life of 20 Years Age is equal to $32\text{-}\frac{7}{8}$. Nay, I may go farther, and aſſert, that the Caſe is exactly the ſame, both with Regard to the Years or Chances loſt in that Time, and the Years or Chances remaining at the End of it, even where we uſe Dr. *Halley*'s, or this Author's Rule for computing the Odds or Chances that a Perſon of ten Years Age does or does not live to the Age of 20 Years, and this too, whether we make ſuch Computation by the whole Period of ten Years collectively, or by each Year ſingly and ſeparately. Conſequently, if a Perſon of ten Years Age has at that Time an even Chance to a Term of 35 Years, which is a Point admitted on all Hands, and in living ten Years loſes no more than $2\frac{7}{8}$ of thoſe Years, or Chances, the Years or Chances remaining muſt of Neceſſity be $32\frac{1}{8}$, and his Life at 20 Years Age be equal to that Term: And therefore the even Chance of Vitality for a Life of 20, computed by Dr. *Halley*'s and this Author's Rule, and ſtated as equal to 27 Years only, is very far wide of the Mark.—

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The Upſhot of the Matter is this; that in ſtating theſe Chances by Dr. *Halley*'s, or this Writer's Rule, which is the ſame, the Decreaſe of them, in moſt Inſtances, eſpecially in the younger Part of Life, is the greateſt when the Number of Perſons dying is proportionably loweſt, and the Decreaſe the leaſt, when the Number dying is higheſt; which is directly contrary to the Truth, for no Man, ſurely, can conceive that theſe Chances ſhould ſink 8 Years in the Period between 10 and 20, in which die only 31, out of 490, that is, at an Average, three only in each Year, and one over in the Whole; and yet, that they ſhould ſink no more than three Years in the Period between 40 and 50, in which there die 90 out of 294, that is, at an Average nine in each Year; and at the ſame Time admit, and it muſt be admitted, that the Number of Perſons dying is the Meaſure of ſuch Decreaſe.

In conſtructing the Tables for theſe Chances of Vitality, and the Value of Annuities on a ſingle Life depending thereon, I made Uſe both of the Government Annuities, and *London* Bills of Mortality, as beforementiond but by the Method laid down by myſelf; and choſe here to compute thoſe Chances by Periods of ten Yeas firſt, and then divide each Period into ſingle Years at an Average, rather than to compute them by ſingle Years, in Regard that in ſom one Year or other the Number of Perſons dying in that Year might much exceed or fall ſhort of the Number dying in the precedent or ſubſequent Year, merely by

by Accident, from the greater or leſs Salubrity of the Air in one Year than another, and this appears to be frequently the Caſe in the Nominees in the Government Annuities. As for the Chances of the intermediate Years within each Period, I interpolated them by an Equation, as near as coud be done, by keeping within the Compaſs of an eighth Part of a Year, and that there I ſuppoſe is as near the Mark as is neceſſary in theſe Matters, or can well be put in Practice. To this I have added, and inſerted in Table N. IX. a Scheme of the Chances of Vitality for the City of *London*, conſtructed, according to my own Method, from the Bills of Mortality there; and, on comparing it with the Chances in the general Tables, the Difference does not ſeem ſo great, but that in Practice, in moſt Caſes, a Man may ſafely enough make Uſe of either of them as he likes beſt: And, tho' I have not here added the Value of an Annuity on any of theſe Chances of Life for *London*, yet that Defect is eaſily ſupplied by one or other of the precedent Tables.—

The Value of Annuities on a Life given in the general Tables, and that which will ariſe on a Computation made by Table, N. IX, will at firſt View and to ſom Perſons ſeem very high; but, for my own Part, I am fully perſuaded, and others, perhaps, when they have well conſiderd what has been here offerd to prove it, may be induced to think ſo too; eſpecially if they admit that the Chances or Probabilities of the Duration of Life are in the Tables rightly eſtimated. Where

a great Number of Annuities is granted to Variety of Perſons on their Lives, a Variety of Chances muſt in the Event neceſſarily come out, and I believe it will be found; that the Eſtimate here made, at an Average, is a fair and reaſonable one, on the Whole: And on ſingle Grants to ſingle Perſons there's no Way to form a Judgment of the Value of them, but by accumulating a great many Grants together, and taking a Medium. And in this my Opinion, that I have not overvalued theſe Annuities for a Life, I am very much confirmd by a late and recent Inſtance, and which will not ſoon be forgot by ſom People, ſince they have terribly ſufferd by not well conſidering or ill calculating the Value of ſuch ſort of Eſtates; and will be a Caution, at leaſt, to Perſons to be on their Guard in Tranſactions of this Nature: *Felix quem faciunt aliena Pericula cautum.*

ADDENDA.

ADDENDA.

AFTER writing the preceding Diſcourſe, and indeed ſince it was prepared and ready for the Preſs, was put into my Hands Mr. *De Moivre*'s Treatiſe on this Subject, the third Edition; and having no convenient Opportunity of inſerting my Obſervations upon it in the Body of my Book, and as his Character challenges a ſpecial Attention, I hope I ſhall be excuſed, if I give my Thoughts upon it in this Place; and they are, in ſhort, that the Rule which he propoſes for inveſtigating the Value of Annuities for a ſingle Life is a wrong one, and all the Tables which he has conſtructed by that Rule, in every Inſtance, erroneous. My great Objection to this Rule, which I think quite unneceſſary to be inſerted here, is the ſame as I made to Dr. *Halley*'s Rule, and is this; *viz.* that in Fact it does, and neceſſarily muſt, produce ſuch a Sum as the Value of an Annuity for one and the ſame individual Life, as that, on the Variation of Intereſt, the Term to which ſuch Value correſponds ſhall be a different one.

Now the Defect in the Doctor's Rule which produces theſe abſurd Values ariſes from hence, that it directs, that out of the Value of an Annuity

nuity certain for a Year, in each Year of Life and through the whole Life, we are to deduct a Value correſponding to the Chance of Mortality ariſing in each Year reſpectively, or, which is much the ſame Thing, a Value in Proportion to the Degree of Probability that the Life may determin within the Year. Mr. *De Moivre*, in the firſt Edition of his Treatiſe on this Subject, P. 5, *&c.* gives the ſame Direction in Subſtance, tho' in Expreſſion ſomthing variant; and in Perſuance and under this Direction, P. 11, *&c.* he computes the Value of an Annuity for the Life of a Perſon aged 50, and at an Intereſt of 5*l. per Cent.* and concludes that 'tis in Value 10-15. In this his third Edition, which he calls a more full one, he has thought fit to leave out the Whole of this Direction, and enters immediately upon the Solution of that Problem, *viz.* what may be the Value of an Annuity for the Life of a Perſon aged 50, and, after a Calculation in the ſame Method as in his firſt Edition, concludes that 'tis in Value 10-35. But 'tis impoſſible that 10-15 and 10-35 ſhould both be the Value of this Annuity; and it happens very unluckily that neither of them are right, for the exact Value is 10-34. But to let this paſs, for 'tis not material which of the three, or whether any of them, be the real Value; but it may be a proper Queſtion, what might be the Reaſon of omitting, in this his fuller Edition, the Whole of this Direction. Som People, perhaps, will ſolve this Problem by ſaying, that, ſince his firſt Edition, he had diſ-

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coverd that this Direction produced the preposterous Values of Annuities before-mentioned; but that, to be sure, must be an idle Surmise, because, even in this more correct Edition, and in his Tables inserted in it, he has persued this very same Direction. Others will suggest, that on finding this Objection made to his Rule, and this Direction in particular, to which they are confident he was no Stranger; and that it was too clear and strong for him to grapple with, and most certainly it has never been answerd, and probably never will be; he thought it the best Course to remove it out of Sight, as much and as far as he coud.

But be that as it will; since I have already, in my *Essay*, treated of this Matter, *viz.* of the absurd Values produced by the Rule, and have farther reinforced my Objections to it in the preceding Treatise; I shall here content myself with asserting, which indeed is very little more than repeating, that 'tis in Nature impossible that one and the same Life can have one and the same Chance or Probability of Duration for different Terms of Years; for Instance, for 25, 26, or 28 Years, as the Rate of Interest, at which the Value of the Annuity is computed, is or may be varied; or indeed on any other Account, or by any other Means whatsoever. I need therefore only to show that the Rule does in Fact produce such Values; and that is seen by a mere Inspection into the Tables constructed by the Author himself, by the Rule, and there they stand thus.

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The Annuity for the Life of a Perſon of 10 Years Age, at 4 *l. per Cent.* is in Value 16-88, which is equal to a Term of 28 $\frac{1}{8}$—of the ſame Age, at 5 *l. per Cent.* is 14-60, which is equal to a Term of 26 $\frac{1}{8}$—and of the ſame Age, at 6 *l. per Cent.* is 12-84, which is equal to a Term of 25 $\frac{1}{4}$: Now I ſay, and I believe moſt of Mankind will concur with me, if a Perſon of 10 Years Age has a Chance or Probability to live only 25 Years and $\frac{1}{4}$, that 'tis not poſſible he ſhould have the ſame Chance or Probability to live to the Age of 26 $\frac{1}{8}$, or of 28 $\frac{1}{8}$.

If we carry this Matter a Step farther, and compute by this Author's Rule what is the Value and Term correſponding on a like Life, at 3 *l. per Cent.* we ſhall find it to be 19-87, which is equal to a Term of 30 $\frac{1}{4}$; and 'tis Matter of ſom Wonderment that this Author, in this his Edition of the Year 1750, has not inſerted in his Tables the Value of Annuities for a Life at that Rate of Intereſt. Since the Government has reduced the Intereſt on public Securities, which in Stock and Annuities amount to more than fifty Millions, from 4 to 3 $\frac{1}{2}$ for ſeven Years, and from thence to 3; and as the Proſpect that the common Price of Money will be at that Rate ſeems to be no very diſtant one; I ſuppoſe, if private Perſons, public Companies, or the Government, were diſpoſed to grant Annuities for Life to any Number of Perſons, that they woud hardly think it prudent to charge their Eſtates or Income with an Intereſt exceeding 3 *l. per Cent.* eſpecially on Securities

Securities not redeemable. On this Account I ſhoud have thought that ſuch a Table, now at leaſt, woud have been very proper, and ſo I have practiſed myſelf; but this Author, perhaps, might be apprehenſive that ſuch a Table woud have expoſed the Error of his Rule too nakedly; for in Truth the Difference between the Term on a Life of 10 Years Age at an Intereſt of 3*l.* and the Term on the ſame Life Intereſt at 6*l.* is five Years and an Half: However, the Tables which he has exhibited have done it ſufficiently.

To the Evidence from his Tables we muſt add, that the Author himſelf in the Body of the Book, in this laſt Edition, has inſerted ſomthing which puts this Matter beyond all Doubt: And, ſince he has been ſo good as to have brought in ſo ſtrong a Reinforcement to ſupport my main Objection, I ſhoud be uncivil, if I did not acknowledge and take Notice of his Supply. Let us read them. In *p.* the 22*d*, he gives us this Problem. *Suppoſing three equal Lives of any given Age, for Inſtance* 30, *and that, upon the Failing of any one of them, that Life ſhall be immediately replaced* (which by the Way has ſomthing of the impoſſible in it) *and I then receive a Sum I agreed upon, and that to Perpetuity for me and my Heirs: What is the preſent Value of that Expectation, and at what Intervals of Time, one with another, may I expect to receive that Sum.*— I have ſet forth the Problem at large, becauſe the ſubſequent Remark which he makes, on which I lay my Finger, becomes thereby more plain

plain and intelligible.— The firſt Queſtion is, *viz.* what is the Value of ſuch Expectancy, after having ſuppoſed that the Sum to be paid on theſe Contingencies was an 100 *l.* he reſolves that the preſent Value is—162 *l.*—Now I very much doubt the Juſtice of this Solution; for the Sum produced differs conſiderably from the Sum produced by the Method which I uſe to compute the Value of ſuch Expectancies, and I may be confident that my Method is a right one, becauſe 'tis founded on the Reaſon of the Thing, and might be confirmd by Demonſtration, and on a Computation in my Manner, if we ſuppoſed theſe Inſtalments to incur once in ten Years for ever, and he ſuppoſes them to incur in $9\frac{1}{4}$, the Value of this Expectancy comes out to be 181-16-3-1, and near a Farthing more.

But let the true Value of it be what it will: We will now read his Remark on the ſecond Queſtion, *viz.* at what Intervals of Time, one with another, he might expect to receive that Sum: And after ſolving that Queſtion, in *p.* the 28*th*, he adds—Remark—*It is not to be expected, if the Intereſt of Money was higher than* 5 *per Cent.* which was the Intereſt made made Uſe of in the preceding Calculations, *the Intervals of Time, after which the Renewals are made, woud be the ſame as now; for it will be found, that, as Intereſt is higher, the Intervals will be ſhorter, and, as it is lower, the Intervals will be longer: Yet one might make it an Objection to our Rules, that the Length of Life woud thereby ſeem to depend upon the Rate*

of

of Intereſt. The Author admits then, that 'tis an Objection to his Rules, if they do in Fact make the Length of Life to depend on the Rate of Intereſt; and, whether he admits it or not, moſt certainly it is ſuch—The ſole Queſtion then will be whether a Variation of the Rate of Intereſt in this Caſe does not likewiſe cauſe a Variation in the Length of the Lives, as it does in the Length of the Intervals. He lays it down, that the Intervals of Renewal, or the Space of Time between one Renewal and another, will and muſt vary as the Intereſt of Money varies, on the one Side or the other. From thence I argue thus: If the Intervals of Renewal are made ſhorter or longer (as the Caſe may happen to be) by a Variation of the Rate of Intereſt, then the Duration of the Lives, on which the Intervals depend, muſt be ſhorter or longer in the like Proportion, becauſe the Duration of the Lives is made, and by this Author himſelf is taken for, the Meaſure of theſe Intervals. Or take the Reaſoning thus: The Rate of Intereſt governs the Intervals, the greater or ſhorter Length of the Lives conſtitutes, or rather is one and the ſame Thing as, thoſe Intervals: The Rate of Intereſt therefore governs the Length of the Lives. This Way of Reaſoning is ſure enough right and concluſive; but it may be further added, that this Writer's Tables, drawn by himſelf, and by his own Rule, in the Inſtances beforementioned, do demonſtrate that, in the Caſe of Annuities for a ſingle Life, the Rate of Intereſt governs the Length of Life,

for, as one is varied, the Length of the other is varied with it and by it.

However, ſince this Writer offers at ſomthing which he calls an Anſwer to this Difficulty, let us read; and I chooſe to give it in his own Words, becauſe it has ſomething of the extraordinary in it, *viz.* a Deſire to ſeem to ſay ſomthing, a Study to involve that ſeeming ſomething in obſcure and perplexd Language, and a Care at the ſame Time to ſay nothing to the Purpoſe; which laſt may indeed very eaſily be done in a Matter to which nothing can be ſaid to the Purpoſe. The Words are theſe. *It muſt be obſerved, that the calculating Time imports no more, than that (conſidering the Circumſtances of the Purchaſer and Proprietor of the Lives, in Reſpect to the Rate of Intereſt agreed upon, and the Sum to be given upon the Renewal of a Life or Lives) the Proprietor makes the ſame Advantage of his Money, as if he had agreed with the Purchaſer, that he ſhould pay him a certain Sum of Money at equal Intervals of Time, for redeeming the Riſque, which he the Purchaſer* (read rather *the Proprietor*) *runs of paying that Sum when the Life or Lives drop; but the real Intervals of Time will be ſhown afterwards.* Part of this Sentence I have included in a Parentheſis, ſo that thoſe Words being omitted in the reading, as they may very well be, it may more eaſily be underſtood; and for the Word *Purchaſer* we muſt read *Proprietor:* And then the Subſtance of his Anſwer will be, that the Proprietor of the perpetual Leaſe, as I woud call it, will make the ſame

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Advantage of his Money, if he continues to make his Payments on the Contingency of Lives dropping, as if he made an Agreement to pay the ſame Sum of Money at the End of a determinate Number of Years, equal to the computed Intervals. But is not the Calculation of the Time a juſt one, and the very ſame, whatever was the Import or Views in making it? Do not the Words at the Cloſe of the Sentence, that the real Intervals of Time ſhall be ſhown afterwards, ſtrongly intimate, and ſuppoſe that there is a real Difference in thoſe Intervals, where there is an Alteration in the Intereſt, otherwiſe no Occaſion to ſhow them, nor coud any be ſhown? Does not the Author himſelf aſſert as much, and is not the Fact notoriouſly true? And is it not evident, that theſe Intervals are created by, or are one and the ſame Thing as, the Duration of the Lives, and this Author himſelf meaſures and demonſtrates the Length of the one by the Length of the other? To theſe Queſtions if the Anſwer be in the Affirmative, and in the Affirmative the Anſwer muſt be; it will follow, as clearly and certainly as any Demonſtration in this Writer's Book, that the Length of the Lives as well as of the Intervals is by his Rule made to vary and depend upon the Rate of Intereſt: And thus by one Breath of his Mouth has he blown in Pieces all his fine Piece of Machinery.—

If we carry our Inquiry farther and examine what Effect his Rule has on the Value of Annuities for joint and ſurviving Lives, and whe-

ther it does not there likewiſe make the Duration of Lives depend on the Rate of Intereſt, we ſhall find the Caſe to be the very ſame: And indeed 'tis impoſſible but that it ſhoud have the ſame Influence there as in Annuities for a ſingle Life, for the Values of Annuities for two or more Lives, either joint or ſurviving, are all of them, by an ill-concerted Jumble of Values, formd from the Values of Annuities for a ſingle Life. If this Author had been ſo good as to have given us Tables of the Values of Annuities for theſe Lives, as he has done of thoſe for a ſingle Life, I doubt not but he woud have enabled us to have ſhown in like Manner, that is, out of his own Mouth, that his Values of Annuities for theſe Lives likewiſe had the very ſame Fault: But, ſince we are deprived of an Opportunity of doing it in that Way, I will try what may be done without that Aſſiſtance.

In *p.* the *54th*, third Edition, he gives us this Problem, *viz.* *to find the Expectation of two Joint Lives, that is, the Time which two Lives may expect to continue together in Being*; and then having taken two Lives of the ſame Age with thoſe given in the Solution of his ſecond Problem, *viz.* one of 40 and the other of 50, he ſolves the Queſtion, and ſays, that 13-31 is the Number of Years due to the two Joint Lives. Now if the Expectation or Probability be, that one or other of theſe two Lives will have a Being for that Term, I humbly apprehend that the Annuity too will ſubſiſt for that Term, and at *5 l. per Cent.* be of the Value

Value 9-54. No, by no Means ſays a Learned Advocate of Mr. *De Moivre*; for he has demonſtrated, in his Solution of the ſecond Problem, and the Demonſtration which follows, that the Sum 7-62 is the true Value of the Annuity for theſe two Joint Lives. The Matter under Conſideration here is, whether the Rate of Intereſt does not govern this Value, and this Value govern the Duration of the two Lives, and not whether the Value be the right one or there be a Demonſtration of it, tho' if there were Room for it here, and it were pertinent to the preſent Purpoſe, I coud ſhow there was none; let us ſee to what Term of Years this Value correſponds. If we look into this Author's third Table, which is conſtructed for the Value of Terms for Years at 5 *l. per Cent.* and that is the Intereſt which is made Uſe of in computing the Value of the Annuity for theſe two Joint Lives; we find 7-72 to be the Value of an Annuity certain for a Term of ten Years; ſo that this Annuity is not equal to a Term of ten Years, but muſt determin before that Time, tho' both the two Lives will ſubſiſt and continue for thirteen Years and above.—

As I do not deſign, here, to deprive this Gentleman and his learned Friend of the Pleaſure of their Conceit (and I can call it by no other Name) that there is Demonſtration of this Value, I will admit it to be ſo for the preſent, on a Proviſo nevertheleſs and Condition that they return me the like Civility, and admit that there is Demonſtra-

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tion likewiſe of the 18th Problem beforementiond, and then we ſhall have Demonſtration fighting againſt Demonſtration; for one ſays that the Annuity is in Value 7·62, and the other ſays that it is 9-54, as before computed. Now, if it may be demonſtrated, nay, whether demonſtrated or not, if it be a reaſonable Expectation or Probability, that one or other of theſe two Lives will ſubſiſt for thirteen Years and above, I very much doubt, whether the Learning of Mr. *De Moivre* and his Friends, both put together, will ever convince any Man, unleſs it be a very ſpecial Friend or ſo, that 'tis probable that this Annuity, which determines, when one of the two Lives fails, and not before, will determin in ten Years Time or leſs, and at the ſame be equally probable, that both the Lives will continue in Being thirteen Years and above. *Credat Judæus appella non ego.*

To apply this. Let then the Value of this Annuity be 7-62, or any other, except 9-54, which is the Value when computed on the Probability of the Duration of the Lives; ſince the Rate of Intereſt creates this Value, and this Value determins the Duration of theſe Lives, if that ſame Rate of Intereſt does not govern the Length of the Lives, I ſhoud be glad to be informd what does, ſince 'tis evident that the Expectation or Probability of the Lives does not; for then the Value woud be 9-54, as before computed. But that which will put the Matter beyond all Diſpute will be to vary the Rate of Intereſt, and to calculate the Value of Annuities on two Joint Lives at the

ſeveral

ſeveral Rates of 4, 5, and 6 *per Cent*, and then examining to what Terms thoſe Values do reſpectively correſpond—On an Annuity for two Joint Lives, each of 45 Years, the Values, and the Terms to which they reſpectively correſpond, as I compute them, ſtand thus—

At 4 *per Cent.* Value 8-21, equal to a Term of 10$\frac{1}{8}$
At 5 *per Cent.* Value 7-42, equal to a Term of 9$\frac{1}{2}$
At 6 *per Cent.* Value 6-76, equal to a Term of 8$\frac{3}{8}$

In Mr. *De Moivre*'s Way of computing theſe Values, they differ a little, and may ſtand thus——

At 4 *per Cent.* Value 8-34, equal to a Term of 10$\frac{3}{8}$
At 5 *per Cent.* Value 7-72, equal to a Term of 10
At 6 *per Cent.* Value 7-06, equal to a Term of 9$\frac{1}{2}$

Now let us take which of the two Valuations we pleaſe, 'tis plain and obvious that the Difference in the Rate of Intereſt produces ſuch a Difference in the Value as creates a Difference in the Term to which the Lives are equal, and this is no other than governing the Length of the Duration of the Lives. It will be ſaid, perhaps, that my Valuation is a wrong one, but, if it ſhoud be ſo, yet the Valuation made in Mr. *De Moivre*'s Method juſtifies the Concluſion; and, if Mr. *De Moivre* has demonſtrated that theſe are the true Values of this Annuity at theſe three ſeveral Rates of Intereſt, he has then demonſtrated the Point in Queſtion, *viz.* that by his Rule the Length of the Lives is made to depend on the Rate of Intereſt—I ſay, however, that my Valuation, where 'tis made

made by a Computation on the Values, is a right one, as founded on the Reaſon of the Thing, and confirmd by this Author's Solution of his 18th Problem, where he ſays, that on two Lives of equal Age, the Probability is $\frac{1}{3}$ of the Complement of any one of the two Lives. Now the Method by which I attaind the Values before given by me is a very eaſy and a very obvious one, *viz.* by taking two thirds of the Value of one of the Lives ſingly taken, or, which is the ſame Thing, by taking one third of the Values of both put together. On this Account and in this View, ſom People will be apt to ſurmiſe that Mr. *De Moivre* had the ſame Thing in his View, *viz.* to take two thirds of the Value of one Life, or one third of the Value of both put together, becauſe 'tis conſonant to his Reaſoning on this 18th Problem; but that he has inveloped the Matter in ſuch Clouds of Multiplications, Subtractions, Diviſions, and of Things not ſubject to the ſame Rules of increaſing or decreaſing, that he has made a ſhift to miſs his own Mark: And this moſt certainly he has done, unleſs he diſclaims his own Problem.

But ſom will ſay that on Mr. *De Moivre*'s Side there is Demonſtration of the Value, but I ſay there is no ſuch Thing. His Solution of his ſecond Problem, which produces this Value 7·72, contains no more than this. The Value of an Annuity for two Lives ſingly taken, being given; then follows a Multiplication of theſe two Values together, then a Multiplication of this Product by the ſuppoſed Rate of Intereſt, afterwards a Subtracti on

Subtraction, and laſtly a Diviſion; and the Quotient is ſaid to be the Value of the Annuity. Now this Proceſs does indeed demonſtrate that the laſt remaining Quantity is 7-72: But does no more prove that 'tis the Value of an Annuity for theſe two Joint Lives, than it does, that 'tis the Value of a Mare or a Mule. The Thing which ought to be proved is, that this Proceſs on the given Suppoſitions will produce ſuch a Sum as is the Value of the Annuity, and not merely what is the Sum which the Proceſs will produce, and the Proceſs carries us no farther. Ay, but then follows a Demonſtration; very true, but that ſtill is no more than this, that Letters and Lines are ſubſtituted in the room of Numbers or Figures, and upon an Operation on them that the Value produced on ſuch Operation is one and the ſame, as that produced by the Proceſs, and it would be very ſtrange if it were otherwiſe; but no Proof yet that this Value is the Value of the Annuity, or that the Proceſs in one or the Operation in the other does produce ſuch a Value. But that's not all: For there is, or ſeems to be, a Defect running both through the Solution and Demonſtration, which muſt neceſſarily make them produce a Sum not exactly the true Value. It may indeed produce a Sum which ſhall come ſomthing near the Truth, and he muſt be a very bungling Operator who coud not contrive Matters, ſo as not to be wide of the Mark; but 'tis a Rule in Mathemetics, I think, that if a Man does not hit the Bird in the Eye, as the common ſaying is,

he is as much in the wrong, as if he were hundred Miles off.

The Defect I meant is this. In computing and fixing the Value of theſe two Annuities, when ſingly taken, he lays it down that the Decrements of Life, or of the Probabilities of the Duration of Life, are in Arithmetic Progreſſion: and then proceeds to multiply theſe two Values thus fixd, and then multiplies the Product by the given Rate of Intereſt, *viz.* by 5. The Years or Chances of Life he calls the Complement of Life, and on the whole poſſible Life ſtates them at 86, aſſerting that Life cannot reaſonably be extended beyond 86; and then, theſe two Lives being each of the Age of 45, the Complement or Chances of Life reſting on each of them will be 41: And then ſays, that theſe 41 Chances will decreaſe in Arithmetic Progreſſion, as the Life proceeds in Age. I will admit that the Chances of Life do decreaſe in that Proportion, under the ſame Limitations as he puts upon them; *viz.* that it holds for ſmall Portions of Life, and when the Years are near one to another, but not abſolutely for the whole Life, when taken intire. But then, as to the fixd Values of the Annuities on theſe two Lives, which are each ſtated at 11-14, there can be no Pretence that they likewiſe decreaſe in the like Arithmetic Progreſſion; for, if they did, then at the End of 20 Years and an Half, for Inſtance, when the Complement or Chances of Life are reduced to Half, then the Value of the Annuity for the Life ſhoud be reduced to Half in like Manner; but this

this is notoriouſly untrue and viſible by a meer Inſpection into his Tables of theſe Values, and comparing them, by which it appears that one is 11-14, and the other 7-05; and the Truth of the Caſe is, that the Decreaſe in the Values is in geometric Progreſſion. And then as to the Number 5, the ſuppoſed Rate of Intereſt, in that there's no Decreaſe at all, it being invariably the ſame through the whole poſſible Life; what ſhall we do with this? Why truly I was thinking, ſince we are upon the decreaſing Pin, that the Show, perhaps, woud have a more oſtentatious Appearance, if, inſtead of 5, we ſhoud take 4-9524, which is the preſent Value of 5 *l.* to be paid at the End of the Year; and we know Intereſt is not payable till the End of each Year, and in Fact is ſeldom paid ſo ſoon, nor do the Probabilities of Life decreaſe in their arithmetic Progreſſion, nor the Values in their geometric Proportion, till the End of the Year; and therefore I hold it adviſable, in the next Edition, by all Means to make them ſink, one and all, and all at one Time. If then we ſubſtitute 4-9524 in the Place of 5, there will be an Opportunity of adding ſom new Numerical and Mathematical Decorations; and beſides upon my Word, *aut video, aut vidiſſe puto*, I ſee, or think I have ſeen, and I muſt not ſay more, becauſe I have not computed it, nor deſign ever to compute it, that the Sum forth coming will be nearer the Mark, *viz.* 7-42, for that moſt certainly is the Value when the Computation is made by Values, as is done by him in this Caſe.

Now to blend and mix together, as it were in Hotchpot, two Things which decreaſe in one Proportion with two other Things which decreaſe in another Proportion, and then to throw in a third Thing, which has no Decreaſe at all; and from ſuch heterogenous Ingredients to hope that we may extract the Value of a third Thing, is ſurely an idle and vain Imagination. But if Mr. *De Moivre* thinks otherwiſe, and will adhere to his Demonſtration, he may, for me, and, for the preſent at leaſt, continue in the peaceable Enjoyment of his beloved Mathematical Proofs, and I will content myſelf with the humble Productions of plain common Senſe and Reaſon. I have ſaid at preſent, becauſe the Queſtion here is not ſo much, whether 7-72 or 7-42 be the true Value of the Annuity for theſe two Joint Lives, and for my own Part I am ſatisfied that neither of them be ſo, as whether on a Variation of the Rate of Intereſt, in this Method of computing their Values, there be not ſuch Values produced as make a Variance likewiſe in the Term to which the Lives correſpond, and in the Length of the Lives. And, ſince that Point is evident, I ſhall not purſue the other any farther here, becauſe it tends rather to ſhow that this Author's Method of inveſtigating the Value of Annuities for Joint Lives is erroneous, and the preceding Diſcourſe treats only of the Value of Annuities for a ſingle Life: And one Time or other, if Health and Leiſure will give me Leave, tho' at my Age of Life neither of them are much

to be expected, I may poſſibly reſume the Diſcuſſion of the ſecond Matter.

I muſt ſay a few Words on the Subject of Annuities for two or more Lives and the Survivor, and I need to ſay very few, becauſe the Point here is clear, that the Length of the Lives is by the Rule made to depend on the Rate of Intereſt. On computing, by the Rule, the Value of an Annuity for three Lives and the Survivor, all of the Age of 12, and at different Rates of Intereſt, the Values, and the Term to which ſuch Values correſpond, ſtand thus—

At 4 *per Cent.* Value 23-01, equal to a Term of 65
At 5 *per Cent.* Value 19-05, equal to a Term of 63
At 6 *per Cent.* Value 16-19, equal to a Term of 61
At 7 *per Cent.* Value 13-92, equal to a Term of 58-2
At 8 *per Cent.* Value 12-29, equal to a Term of 56

I do not affirm that theſe Values, all of them, are exactly true; however, they are near enough the Truth to anſwer the Purpoſe for which they are here inſerted, *viz.* to ſhow that, whenever we vary the Rate of Intereſt, we find a Variance likewiſe in the Term to which the Lives are equal; that is, that the Rate of Intereſt governs and determins the Length of the Lives; and it appears here that, between the higheſt and loweſt Intereſt, the Difference in the Term is nine Years. This wants no farther Comment, ſo I ſhall add only a ſhort Obſervation, tho', ſhort as it is, it trenches deep into the Scheme, and all the Rules which Mr. *De Moivre* has contrived for inveſtigating the Value

Value of Annuities for one or more Lives, of any Kind, or in any Shape; and my Remark is this. On computing, by his Scheme and his Rules, the Value of Annuities for a ſingle Life, and for two or more Joint Lives, the Values come out to be ſuch as correſpond to a Term below the even Chance, or Probability of the Duration of the Life or Lives: And on the contrary, on computing the Value of Annuities for two or more Lives and the Survivor, the Values are always ſuch as correſpond to a Term above the even Chance, or Probability of the Duration of the Lives, or the Survivor of them. And I will ſay here, as I ſaid in my *Eſſay*, on much the like Occurrence, that this is a Contradiction to common Senſe and Reaſon, and leave it to Mr. *De Moivre* to reconcile it if he can; but I bar all Proofs from Mathematical Schemes, unleſs he take ſpecial Care that his Data and Poſtulate are all right and clear.

TABLES

Of the VALUE of

ANNUITIES

AND

LEASES certain,

For Years, and for a ſingle Life, at every Age of Life.

At the the Rates of 3, 4, 5, and 6 *per Cent.* Intereſt.

TABLE I.

Showing the Value of Annuities and Leaſes certain, for any Term of Years, in Years, and in the eighth Part, and hundredth Part of a Year. Intereſt, three *per Cent.*

Term of Years.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100
1	00-7-97	26	17-6-86	51	25-6-86
2	01-7-91	27	18-2-31	60	27-5-68
3	02-6-83	28	18-6-75	70	29-1-12
4	03-5-61	29	19-1-17	80	30-1-21
5	04-4-58	30	19-4-58	90	31——
6	05-3-42	31	19-7-98	100	31-4-60
7	06-2-24	32	20-3-37	Fee	33-2-33
8	07---03	33	20-6-75		
9	07-6-81	34	21-1-12		
10	08-4-56	35	21-3-48		
11	09-2-29	36	21-6-83		
12	10——	37	22-1-17		
13	10-5-67	38	22-4-19		
14	11-2-33	39	22-6-79		
15	11-7-97	40	23—11		
16	12-4-59	41	23-3-42		
17	13-1-19	42	23-5-71		
18	13-6-78	43	24——		
19	14-2-35	44	24-2-27		
20	14-7-89	45	24-4-53		
21	15-3-42	46	24-6-78		
22	15-7-94	47	25—02		
23	16-3-44	48	25-2-25		
24	16-7-92	49	25-3-47		
25	17-3-40	50	25-5-67		

TABLE II.

Showing the Term to which Annuities and Leaſes certain, for a ſingle Life, are computed to be equal; and the Value of ſuch Annuities and Leaſes, in Years, and the eighth Part of a Year. Intereſt, three *per Cent.*

Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.	Age.	Term Y.8th.	Val. in Y.8th.
1	25--	17-3	26	27-4	18-4	51	15-3	12-1
2	28--	18-6	27	26-7	18-2	52	15--	11-7
3	31--	19-7	28	26-2	17-7	53	14-4	11-5
4	33--	20-6	29	25-5	17-5	54	14--	11-2
5	34-4	21-2	30	25--	17-3	55	13-4	11--
6	35--	21-3	31	24-4	17-1	56	13--	10-5
7	35--	21-3	32	24--	16-7	57	12-4	10-3
8	35--	21-3	33	23-4	16-5	58	12--	10--
9	35--	21-3	34	23--	16-3	59	11-4	9-5
10	35--	21-3	35	22-4	16-1	60	11--	9-2
11	34-4	21-2	36	22--	15-7	61	10-4	8-7
12	34--	21-1	37	21-4	15-5	62	10--	8-4
13	33-5	20-7	38	21--	15-3	63	9-4	8-1
14	33-2	20-6	39	20-4	15-1	64	9--	7-6
15	32-7	20-5	40	20--	14-7	65	8-4	7-3
16	32-4	20-4	41	19-4	14-5	66	8--	7--
17	32-1	20-3	42	19--	14-3	67	7-3	6-4
18	31-6	20-2	43	18-4	14--	68	6-6	6--
19	31-3	20-1	44	18--	13-6	69	6-1	5-4
20	31--	19-7	45	17-5	13-4	70	5-4	5--
21	30-4	19-6	46	17-2	13-2	75	4-2	3-7
22	30--	19-5	47	16-7	13--	80	3--	2-6
23	29-3	19-3	48	16-4	12-7	85	2--	1-7
24	28-6	19--	49	16-1	12-5	90	1--	0-7
25	28-1	18-6	50	15-6	12-3	*&c.*		

TABLE III.

Showing the Value of Annuities and Leaſes certain, for any Term of Years; in Years, and in the eighth Part, and hundredth Part of a Year. Intereſt, four *per Cent.*

Term of Years.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100
1	00-7-96	26	15-7-97	51	21-4-60
2	01-7-88	27	16-2-31	60	22-4-61
3	02-6-78	28	16-5-64	70	23-3-39
4	03-5-62	29	16-7-97	80	23-7-90
5	04-3-44	30	17-2-29	90	24-2-30
6	05-1-23	31	17-4-57	100	24-4-50
7	06——	32	17-6-85	Fee	25——
8	06-5-72	33	18—11		
9	07-3-42	34	18-3-37		
10	08—10	35	18-5-62		
11	08-6-75	36	18-6-86		
12	09-3-37	37	19—10		
13	09-7-97	38	19-2-33		
14	10-4-55	39	19-4-56		
15	11—10	40	19-6-78		
16	11-5-64	41	20——		
17	12-1-15	42	20-1-20		
18	12-5-64	43	20-3-39		
19	13-1-12	44	20-4-57		
20	13-4-59	45	20-5-73		
21	14—01	46	20-7-90		
22	14-3-46	47	21—05		
23	14-6-84	48	21-1-20		
24	15-2-25	49	21-2-34		
25	15-4-61	50	21-3-47		

TABLE IV.

Showing the Term to which Annuities and Leaſes certain, for a ſingle Life, are computed to be equal; and the Value of ſuch Annuities and Leaſes, in Years, and the eighth Part of a Year. Intereſt, four *per Cent.*

Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.
1	25--	15-3	26	27-4	16-3	51	15-3	11-2
2	28--	16-5	27	26-7	16-2	52	15--	11--
3	31--	17-4	28	26-2	16--	53	14-4	10-6
4	33--	18--	29	25-5	15-6	54	14--	10-4
5	34-4	18-4	30	25--	15-4	55	13-4	10-2
6	35--	18-5	31	24-4	15-3	56	13--	9-7
7	35--	18-5	32	24--	15-2	57	12-4	9-5
8	35--	18-5	33	23-4	15--	58	12--	9-3
9	35--	18-5	34	23--	14-6	59	11-4	9--
10	35--	18-5	35	22-4	14-5	60	11--	8-6
11	34-4	18-4	36	22--	14-3	61	10-4	8-3
12	34--	18-3	37	21-4	14-2	62	10--	8--
13	33-5	18-2	38	21--	14--	63	9-4	7-6
14	33-2	18-1	39	20-4	13-6	64	9--	7-3
15	32-7	18--	40	20--	13-4	65	8-4	7--
16	32-4	18--	41	19-4	13-2	66	8--	6-5
17	32-1	17-7	42	19--	13-1	67	7-3	6-2
18	31-6	17-6	43	18-4	12-7	68	6-6	5-6
19	31-3	17-5	44	18--	12-5	69	6-1	5-2
20	31--	17-4	45	17-5	12-3	70	5-4	4-6
21	30-4	17-3	46	17-2	12-2	75	4-2	3-6
22	30--	17-2	47	16-7	12--	80	3--	2-6
23	29-3	17-1	48	16-4	11-7	85	2--	1-7
24	28-6	16-7	49	16-1	11-5	90	1--	0-7
25	28-1	16-5	50	15-6	11-4	*&c.*		

TABLE V.

Showing the Value of Annuities and Leaſes certain, for any Term of Years; in Years, and in the eighth Part, and hundredth Part of a Year. Intereſt, five *per Cent.*

Term of Years.	Value in th th. Y. 8 100	Term	Value in th th. Y. 8 100	Term	Value in th th. Y. 8 100
1	00-7-95	26	14-3-37	51	18-2-33
2	01-6-86	27	14-5-64	60	18-7-93
3	02-5-72	28	14-7-89	70	19-2-34
4	03-4-54	29	15-1-13	80	19-4-59
5	04-2-33	30	15-2-36	90	19-6-75
6	05—07	31	15-4-59	100	19-6-85
7	05-6-79	32	15-6-80	Fee	20——
8	06-3-46	33	16——		
9	07—11	34	16-1-19		
10	07-5-71	35	16-3-37		
11	08-2-31	36	16-4-54		
12	08-7-87	37	16-5-70		
13	09-3-39	38	16-6-86		
14	09-7-89	39	17—01		
15	10-3-38	40	17-1-15		
16	10-6-84	41	17-2-30		
17	11-2-27	42	17-3-43		
18	11-5-68	43	17-4-53		
19	12——	44	17-5-60		
20	12-3-46	45	17-6-77		
21	12-6-82	46	17-7-88		
22	13-1-16	47	17-7-98		
23	13-3-48	48	18—08		
24	13-6-79	49	18-1-17		
25	14—09	50	18-2-25		

TABLE VI.

Showing the Term to which Annuities and Leaſes certain, for a ſingle Life, are computed to be equal; and the Value of ſuch Annuities and Leaſes, in Years, and the eighth Part of a Year. Intereſt, five *per Cent.*

Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.
1	25--	14--	26	27-4	14-6	51	15-3	10-4
2	28--	14-7	27	26-7	14-5	52	15--	10-3
3	31--	15-4	28	26-2	14-3	53	14-4	10-1
4	33--	16--	29	25-5	14-2	54	14--	9-7
5	34-4	16-2	30	25--	14--	55	13-4	9-5
6	35--	16-3	31	24-4	13-7	56	13--	9-3
7	35--	16-3	32	24--	13-6	57	12-4	9-1
8	35--	16-3	33	23-4	13-5	58	12--	8-7
9	35--	16-3	34	23--	13-4	59	11-4	8-4
10	35--	16-3	35	22-4	13-3	60	11--	8-2
11	34-4	16-2	36	22--	13-1	61	10-4	8--
12	34--	16-1	37	21-4	12-7	62	10--	7-5
13	33-5	16--	38	21--	12-6	63	9-4	7-3
14	33-2	16--	39	20-4	12-5	64	9--	7--
15	34-7	15-7	40	20--	12-3	65	8-4	6-6
16	32-4	15-7	41	19-4	12-2	66	8--	6-3
17	32-1	15-6	42	19--	12--	67	7-3	6--
18	31-6	15-5	43	18-4	11-7	68	6-6	5-4
19	31-3	15-5	44	18--	11-5	69	6-1	5-1
20	31--	15-4	45	17-5	11-4	70	5-4	4-5
21	30-4	15-3	46	17-2	11-3	75	4-2	3-5
22	30--	15-2	47	16-7	11-1	80	3--	2-5
23	29-3	15-1	48	16-4	11--	85	2--	1-6
24	28-6	15--	49	16-1	10-7	90	1--	0-7
25	28-1	14-7	50	15-6	10-5	*&c.*		

TABLE VII.

Showing the Value of Annuities and Leaſes certain, for any Term of Years, in Years, and in the eighth Part, and hundredth Part of a Year. Intereſt, ſix *per Cent.*

Term of Years.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100	Term.	Value in th th. Y. 8 100
1	00-7-94	26	13——	51	15-6-79
2	01-6-83	27	13-1-21	60	16-1-17
3	02-5-67	28	13-3-40	70	16-3-39
4	03-3-46	29	13-4-58	80	16-4-50
5	04-1-21	30	13-6-76	90	16-4-58
6	04-7-92	31	13-7-93	100	16-4-61
7	05-4-58	32	14—09	Fee	16-5-67
8	06-1-21	33	14-1-23		
9	06-6-80	34	14-3-38		
10	07-2-36	35	14-4-51		
11	07-7-88	36	14-5-63		
12	08-3-38	37	14-6-74		
13	08-6-85	38	14-6-84		
14	09-2-29	39	14-7-94		
15	09-5-71	40	15—03		
16	10—10	41	15-1-12		
17	10-3-47	42	15-1-20		
18	10-6-83	43	15-2-28		
19	11-1-16	44	15-2-36		
20	11-3-47	45	15-3-43		
21	11-6-76	46	15-4-50		
22	12—03	47	15-4-56		
23	12-2-30	48	15-5-62		
24	12-4-55	49	15-5-68		
25	12-6-78	50	15-6-74		

TABLE VIII.

Showing the Term to which Annuities and Leaſes certain, for a ſingle Life, are computed to be equal; and the Value of ſuch Annuities and Leaſes, in Years, and the eighth Part of a Year. Intereſt, ſix *per Cent.*

Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.	Age.	Term Y. 8th.	Val. in Y. 8th.
1	25--	12-6	26	27-4	13-2	51	15-3	9-6
2	28--	13-3	27	26-7	13-1	52	15--	9-5
3	31--	13-7	28	26-2	13--	53	14-4	9-4
4	33--	14-1	29	25-5	12-7	54	14--	9-2
5	34-4	14-3	30	25--	12-6	55	13-4	9--
6	35--	14-4	31	24-4	12-5	56	13--	8-6
7	35--	14-4	32	24--	12-4	57	12-4	8-4
8	35--	14-4	33	23-4	12-3	58	12--	8-3
9	35--	14-4	34	23--	12-2	59	11-4	8-1
10	35--	14-4	35	22-4	12-1	60	11--	7-7
11	34-4	14-3	36	22--	12--	61	10-4	7-5
12	34--	14-3	37	21-4	11-7	62	10--	7-2
13	33-5	14-2	38	21--	11-6	63	9-4	7--
14	33-2	14-2	39	20-4	11-5	64	9--	6-6
15	32-7	14-1	40	20--	11-3	65	8-4	6-4
16	32-4	14-1	41	19-4	11-2	66	8--	6-1
17	32-1	14--	42	19--	11--	67	7-3	5-6
18	31-6	14--	43	18-4	10-7	68	6-6	5-3
19	31-3	13-7	44	18--	10-6	69	6-1	5--
20	31--	13-7	45	17-5	10-5	70	5-4	4-5
21	30-4	13-6	46	17-2	10-4	75	4-2	3-4
22	30--	13-6	47	16-7	10-3	80	3--	2-5
23	29-3	13-5	48	16-4	10-2	85	2--	1-6
24	28-6	13-4	49	16-1	10-1	90	1--	0-7
25	28-1	13-3	50	15-6	10--	*&c.*		

TABLE IX.

Showing the Term in Years, and the eighth Part of a Year, to which Lives of all Ages in the City of *London* are computed to be equal.

Ages.	Y.	8th.	Ages.	Y.	8th.	Ages.	Y.	8th.
1	25	-	26	30	-	51	14	2
2	28	-	27	29	4	52	13	6
3	31	-	28	28	7	53	13	2
4	33	-	29	28	2	54	12	6
5	34	4	30	27	5	55	12	2
6	35	-	31	27	-	56	11	6
7	35	-	32	26	3	57	11	2
8	35	-	33	25	6	58	10	6
9	35	-	34	25	1	59	10	2
10	35	-	35	24	4	60	9	6
11	34	5	36	23	7	61	9	2
12	34	2	37	23	2	62	8	6
13	34	-	38	22	5	63	8	2
14	33	6	39	21	7	64	7	6
15	33	5	40	21	1	65	7	3
16	33	4	41	20	3	66	7	-
17	33	3	42	19	6	67	6	5
18	33	2	43	19	1	68	6	2
19	33	1	44	18	4	69	5	7
20	32	7	45	17	7	70	5	4
21	32	4	46	17	2	80	2	7
22	32	-	47	16	5	90	1	1
23	31	4	48	16	-	*&c.*		
24	31	-	49	15	3			
25	30	4	50	14	6			

TWO SCHEMES:

The FIRST,

Showing the Proportion in which the ſuppoſed 35 Years or Chances of Vitality, on the Life of a Perſon of ten Years Age, decreaſe in each Period of ten Years; as ſtated by Mr. *Hodgſon* on the *London* Bills of Mortality, *p.* 9, *&c.*

The SECOND,

As computed by Mr. *Lee* on the ſame *London* Bills of Mortality.

SCHEME I.

Showing the Proportion in which the ſuppoſed 35 Years or Chances of Vitality, on the Life of a Perſon of ten Years Age, decreaſe in each Period of ten Years; as ſtated by Mr. *Hodgſon* on the *London* Bills of Mortality, *p.* 9, *&c.*

From			Y.	8th.
10	to	20	8	-
20	to	30	5	4
30	to	40	4	-
40	to	50	3	-
50	to	60	3	4
60	to	70	2	4
70	to	80	3	4
80	to	90	2	-
90	and	upwards	3	-
			35	-

SCHEME II.

Showing the Proportion in which the ſuppoſed 35 Years or Chances of Vitality, on the Life of a Perſon of ten Years Age, decreaſe in each Period of ten Years; as computed by Mr. *Lee* on the ſame *London* Bills of Mortality.

From			Y.		8th.
10	to	20	2	——	1
20	to	30	5	——	2
30	to	40	6	——	4
40	to	50	6	——	3
50	to	60	5	——	-
60	to	70	4	——	3
70	to	80	2	——	6
80	to	90	1	——	4
90 and upwards			1	——	1
			35	——	-

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